Homework 2 – Number Theory and Counting

Exercise 1. Calculate the greatest common divisors of numbers shown below and express this value in the form of the Bézout identity.

(a) $\gcd(12, 17)$ (b) $\gcd(27, 12)$ (c) $\gcd(65, 5)$ (d) $\gcd(10, 27)$

Exercise 2. Answer the questions below.

(a) Which integers are congruent to 3 mod 7?

(b) List integers in the equivalence class of 5 mod 10?

Exercise 3. Calculate

(a) 3 mod 5 (b) 5 mod 3 (c) 12 mod 3 (d) 7 mod 4

(e) -5 mod 8 (f) -4 mod 11 (g) 6^{-1} mod 7 (h) 2^{-1} mod 6

Exercise 4. Solve for x. If the equation is not solvable, provide a justification for it.

(a) $x + 12 \equiv 7 \pmod{15}$ (b) $4x \equiv 3 \pmod 7$

(c) $15x + 12 \equiv 21 \pmod{27}$ (d) $8x \equiv 3 \pmod{28}$

Exercise 5. Solve for x. If the system is not solvable, provide a justification for it.

(a) $\begin{cases} 5a + b \equiv 0 \pmod 8 \\ 2a + b \equiv 1 \pmod 8 \end{cases}$ (b) $\begin{cases} 3a + b \equiv 6 \pmod 7 \\ 6a + b \equiv 4 \pmod 7 \end{cases}$

(c) $\begin{cases} 5a + b \equiv 4 \pmod 6 \\ 3a + b \equiv 5 \pmod 6 \end{cases}$ (d) $\begin{cases} 9a + b \equiv 1 \pmod{10} \\ 5a + b \equiv 5 \pmod{10} \end{cases}$

Exercise 6. Solve for x.

(a) $\begin{cases} x \equiv 2 \pmod 3 \\ x \equiv 4 \pmod 5 \end{cases}$ (b) $\begin{cases} x \equiv 3 \pmod 4 \\ x \equiv 7 \pmod 9 \end{cases}$

(c) $\begin{cases} x \equiv 3 \pmod 5 \\ x \equiv 5 \pmod 7 \\ x \equiv 6 \pmod 8 \end{cases}$ (d) $\begin{cases} x \equiv 6 \pmod{10} \\ x \equiv 3 \pmod{13} \\ x \equiv 15 \pmod{19} \end{cases}$

Exercise 7. Calculate the value of the Euler's totient function $\varphi(n)$.

(a) $\varphi(11)$ (b) $\varphi(99)$

(c) $\varphi(20)$ (d) $\varphi(540)$

Exercise 8. Andy has 5 toy ships and 6 toy planes. He wants to make an exhibition showing 3 models of one kind and 4 models of the other kind. How many ways there are to pick the exhibition set from his collection?

Exercise 9. How many ways there are to line up n male and $n - 1$ female students for a group photo so that in the resulting arrangement no two males stand side by side?

Exercise 10. Solve the recurrence $A_{n+2} = A_{n+1} + 2A_n + 1$, when $A_0 = 0$, $A_1 = 2$.