Lecture 6
Module I: Model Checking
Topic: Model checking timed transition systems: timed automata

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10.03.2022

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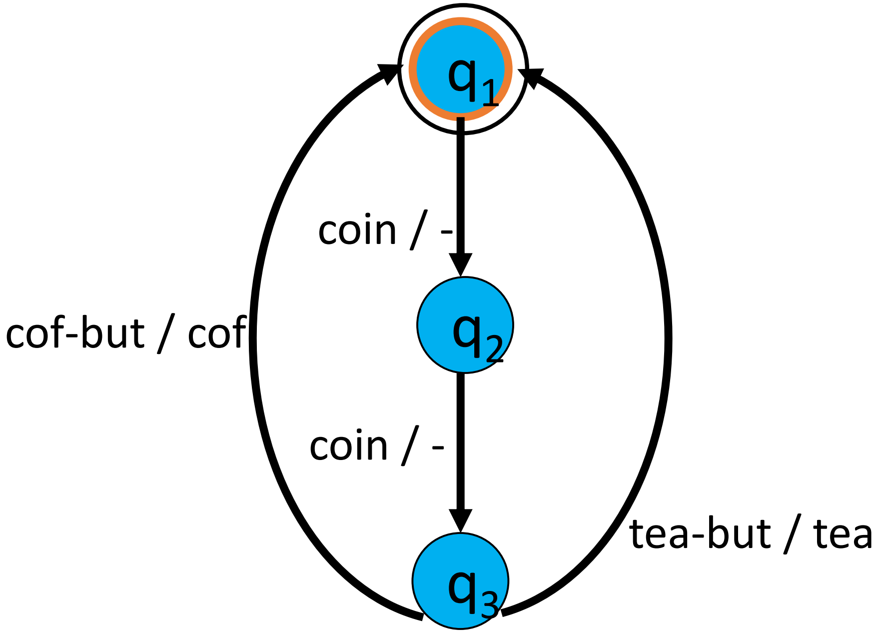
Finite State Machine (Mealy)



condition		effect	
current state	input	output	next state
q_1	coin	-	q_2
q_2	coin	-	q_3
q_3	cof-but	cof	q_1
q_3	tea-but	tea	q_1

Inputs = {cof-but, tea-but, coin}
Outputs = {cof,tea}
States: $\{q_1,q_2,q_3\}$
Initial state = q_1
Transitions= {
$(q_1$, coin, -, $q_2)$,
$(q_2$, coin, -, $q_3)$,
$(q_3$, cof-but, cof, $q_1)$,
$(q_3$, tea-but, tea, $q_1)$
}

Sample run:

$$q_1 \xrightarrow{\text{coin/ -}} q_2 \xrightarrow{\text{coin/-}} q_3 \xrightarrow{\text{cof-but / cof}} q_1 \xrightarrow{\text{coin/ -}}$$

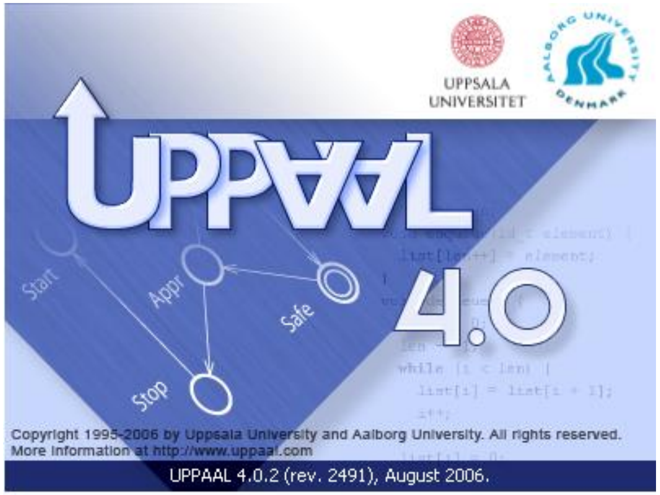
$$q_2 \xrightarrow{\text{coin/ -}} q_3 \xrightarrow{\text{cof-but / cof}} q_1$$

Adding Time

FSM

↓

Timed Automata



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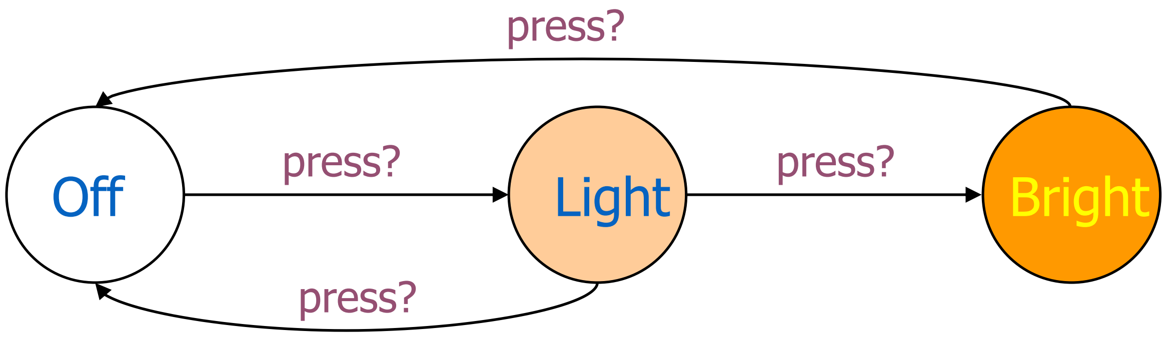
Designing Dumb Light Control

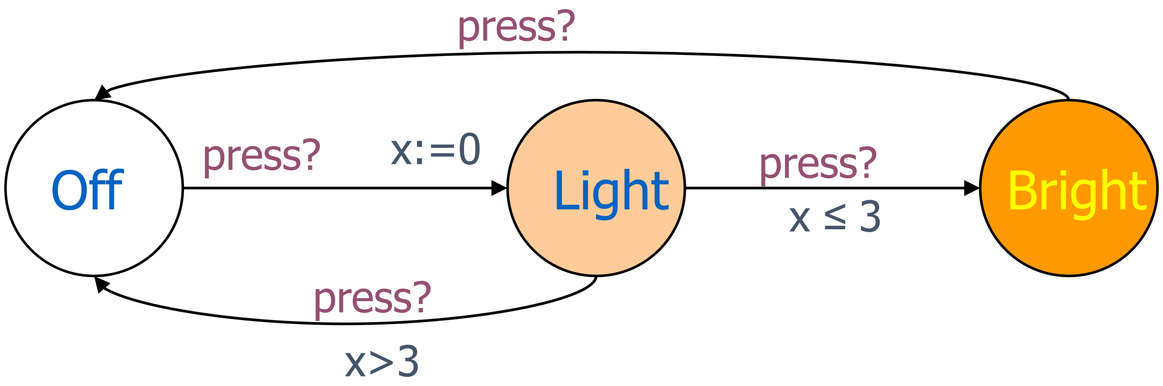
Reguirement: if press is issued twice quickly then the light will get brighter; otherwise the light is turned off.

Solution 1:

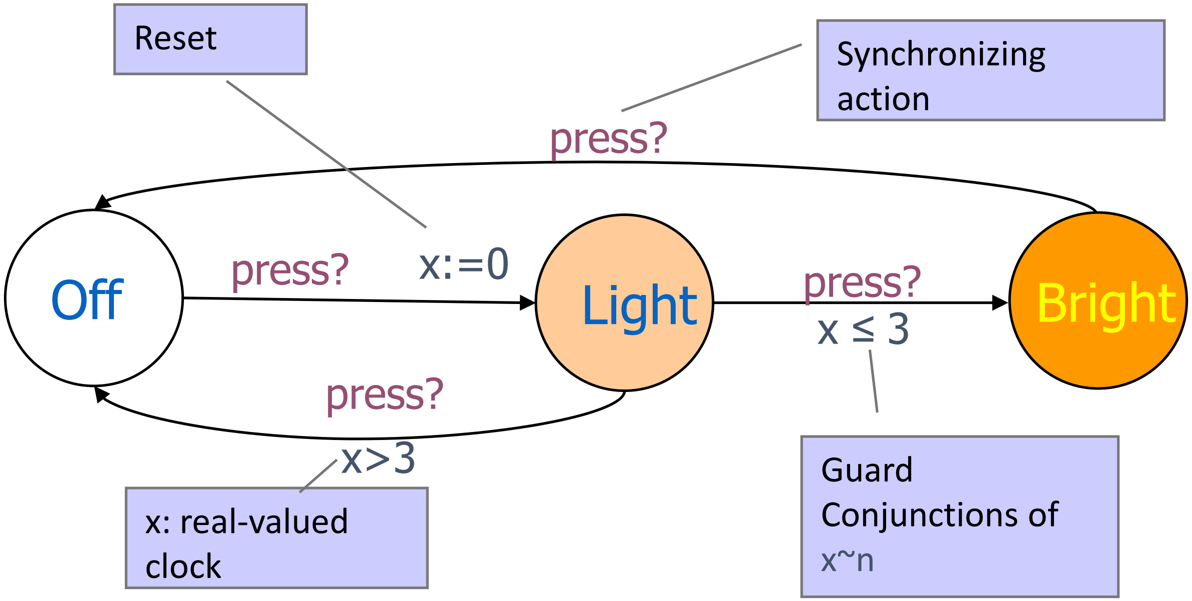


Dumb Light Control

Solution 2: Add real-valued clock x to model the timing requirement *quickly* ≡ $x \leq 3$



Timed Automata



Reset

Synchronizing action

x: real-valued clock

Guard Conjunctions of x~n

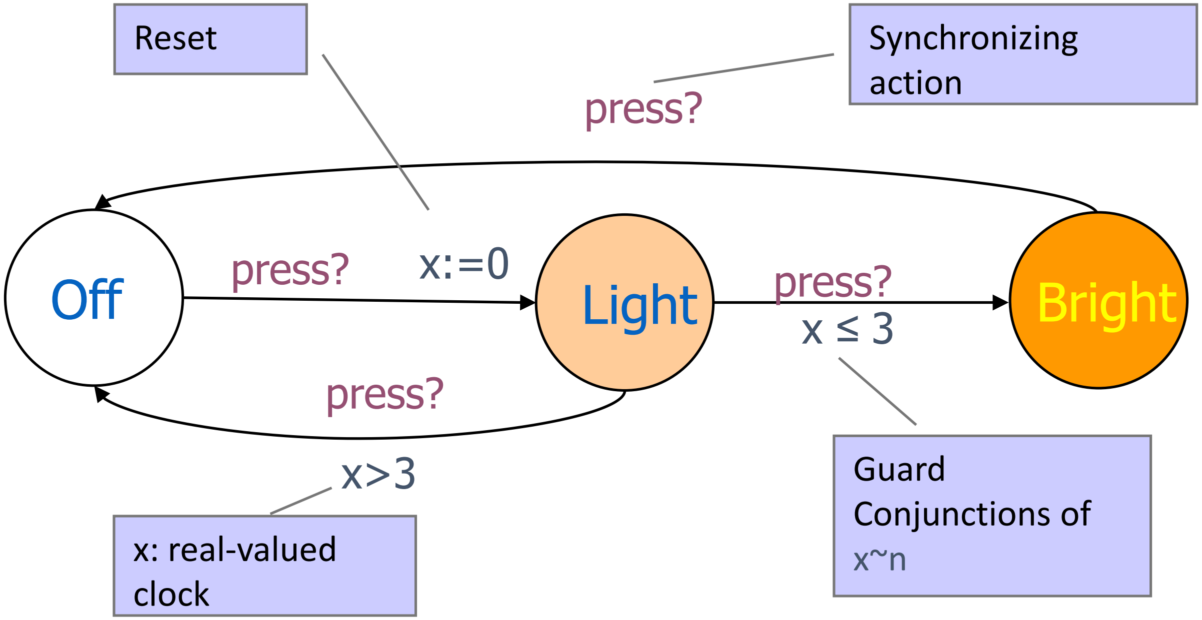
States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

Timed Automata



Reset

Synchronizing action

x: real-valued clock

Guard
Conjunctions of x~n

States:

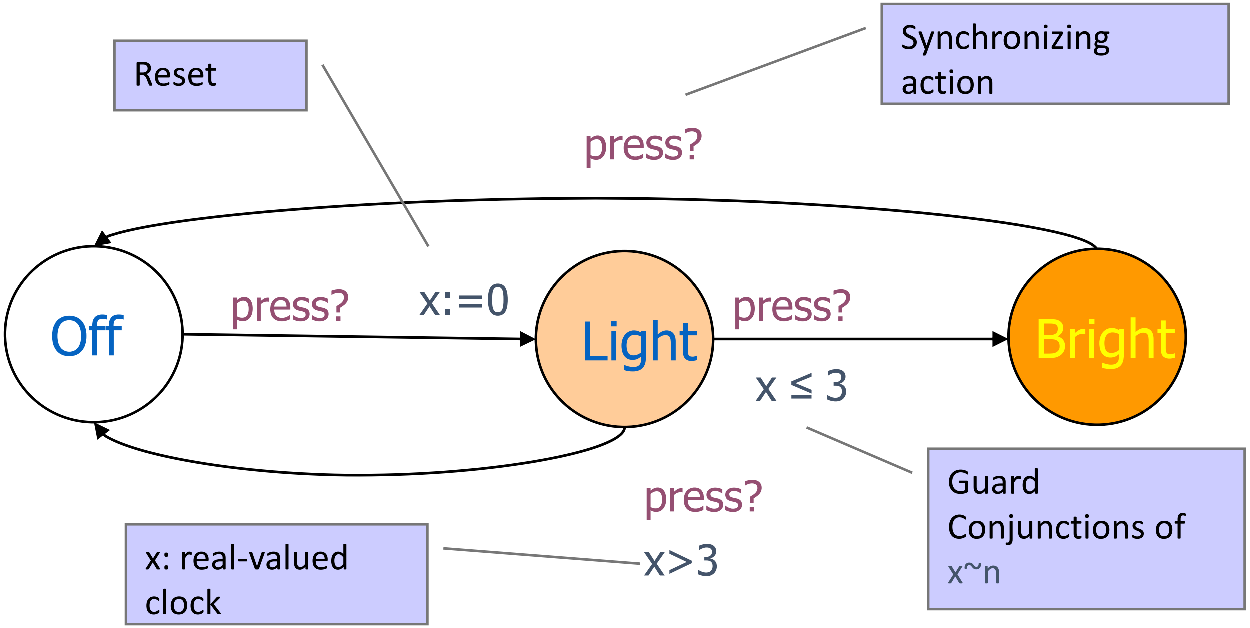
(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

Timed Automata



Reset

Synchronizing action

Guard
Conjunctions of x~n

x: real-valued clock

States:

(location , x=v) where v ∈ **R**

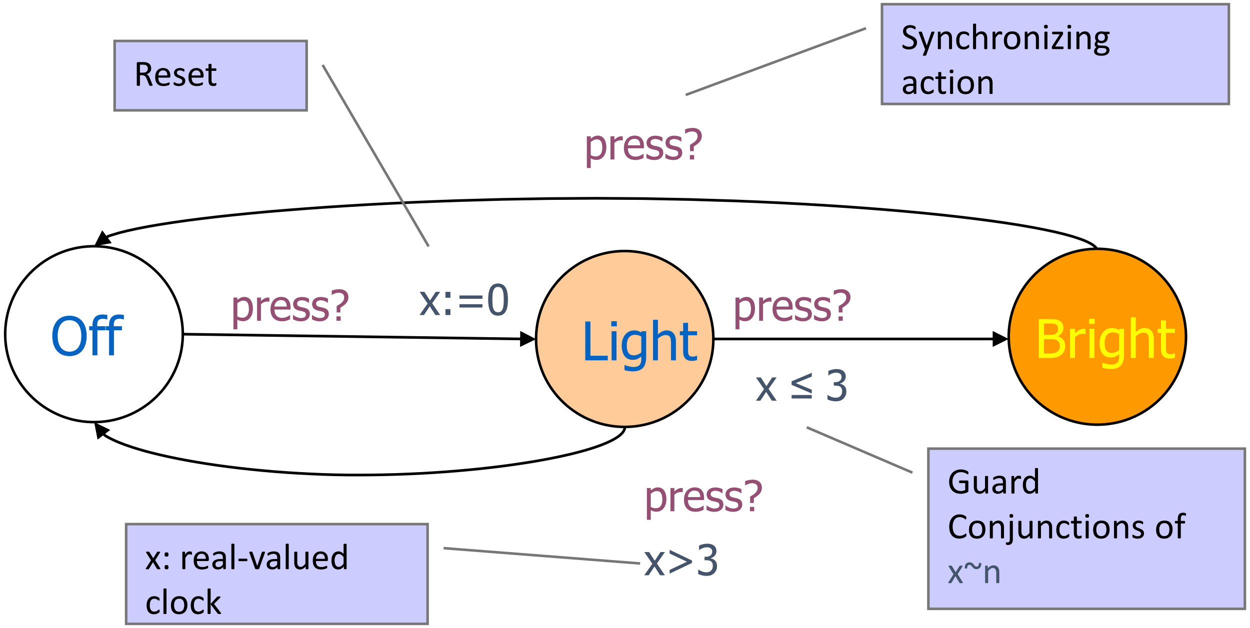
Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

press? → (Light , x=0)

Timed Automata



Reset

Synchronizing action

Guard
Conjunctions of x~n

x: real-valued clock

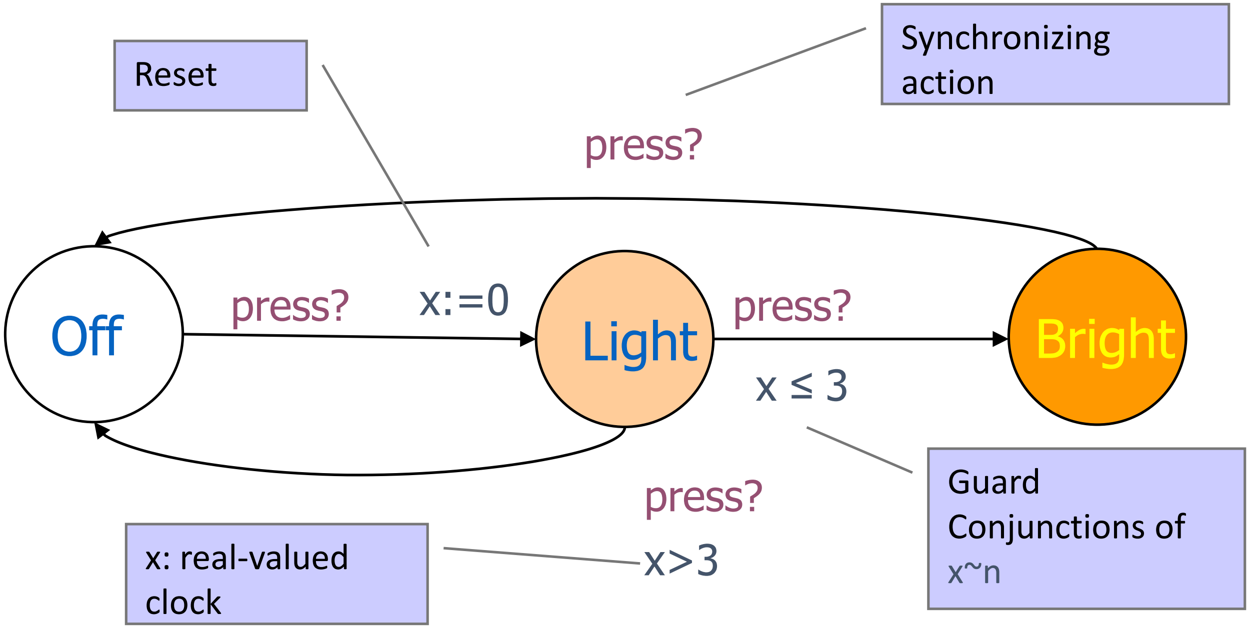
States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)
delay 4.32 → (Off , x=4.32)
press? → (Light , x=0)
delay 2.51 → (Light , x=2.51)

Timed Automata



States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

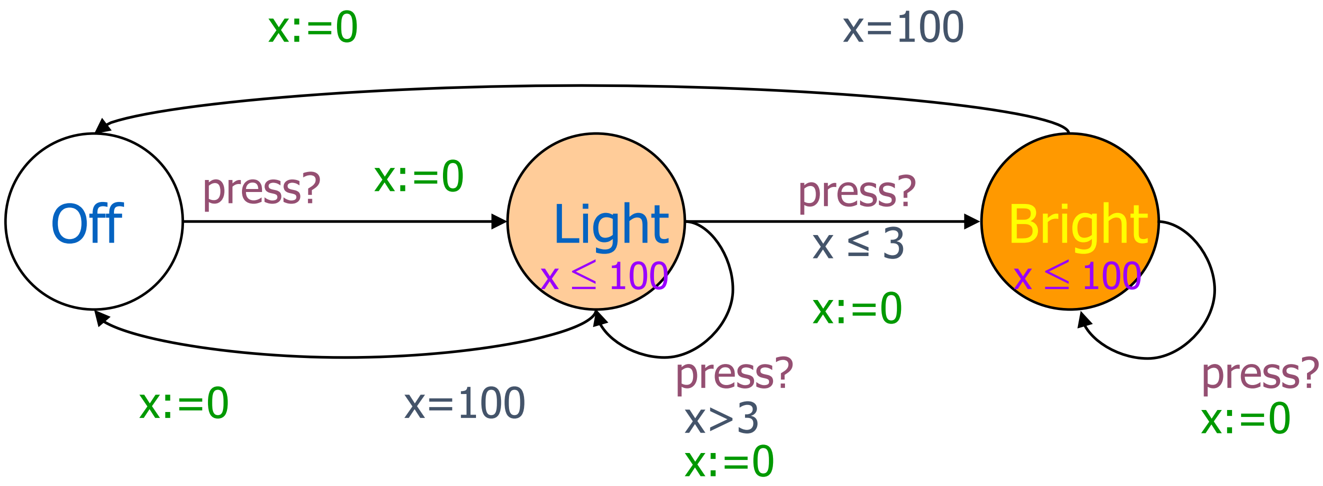
press? → (Light , x=0)

delay 2.51 → (Light , x=2.51)

press? → (Bright , x=2.51)

Intelligent Light Control

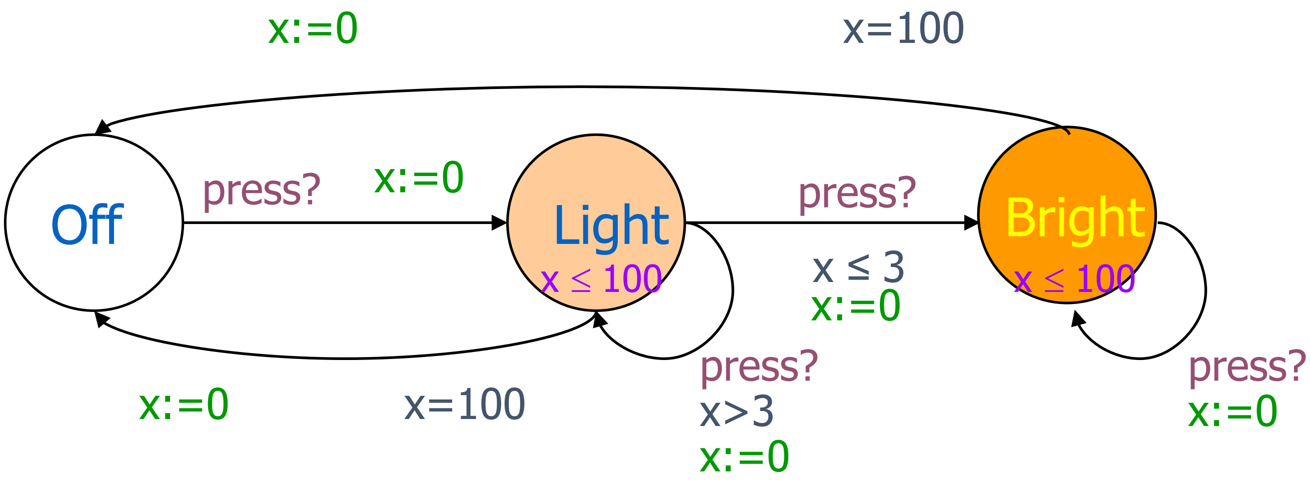
Reguirement: : automatically switch light off after 100 time units



Upper time bound is specified using invariants

Intelligent Light Control

Using Invariants



Transitions:

(Off , x=0)
delay 4.32 → (Off , x=4.32)
press? → (Light , x=0)
delay 4.51 → (Light , x=4.51)
press? → (Light , x=0)
delay 100 → (Light , x=100)
τ → (Off , x=0)

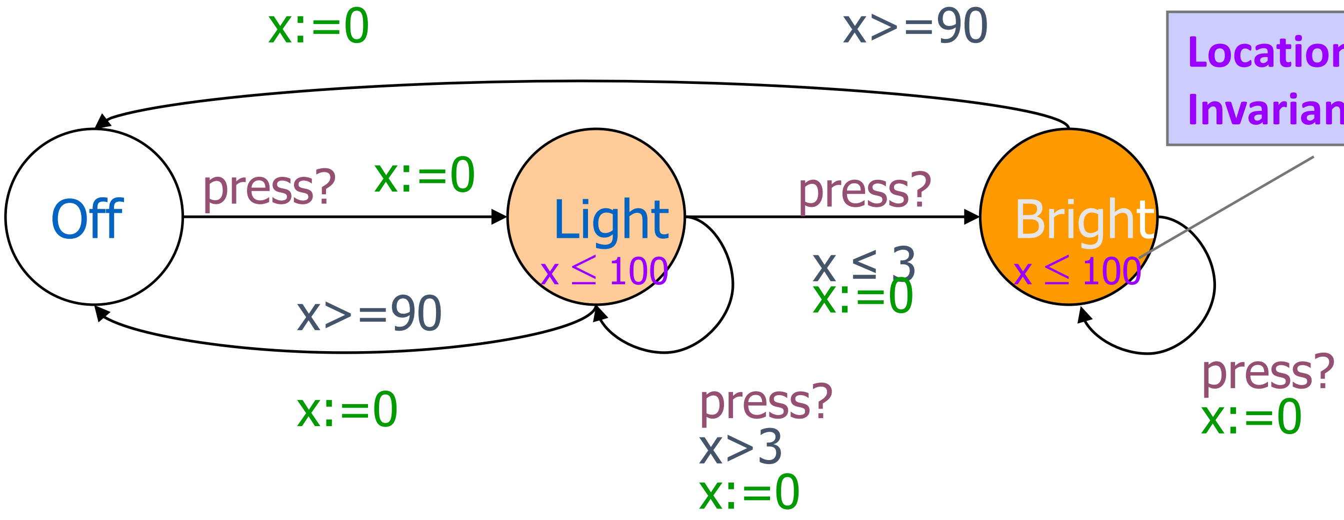
Note:
(Light , x=0) delay 103 → X

Invariants ensures progress

Intelligent Light Control

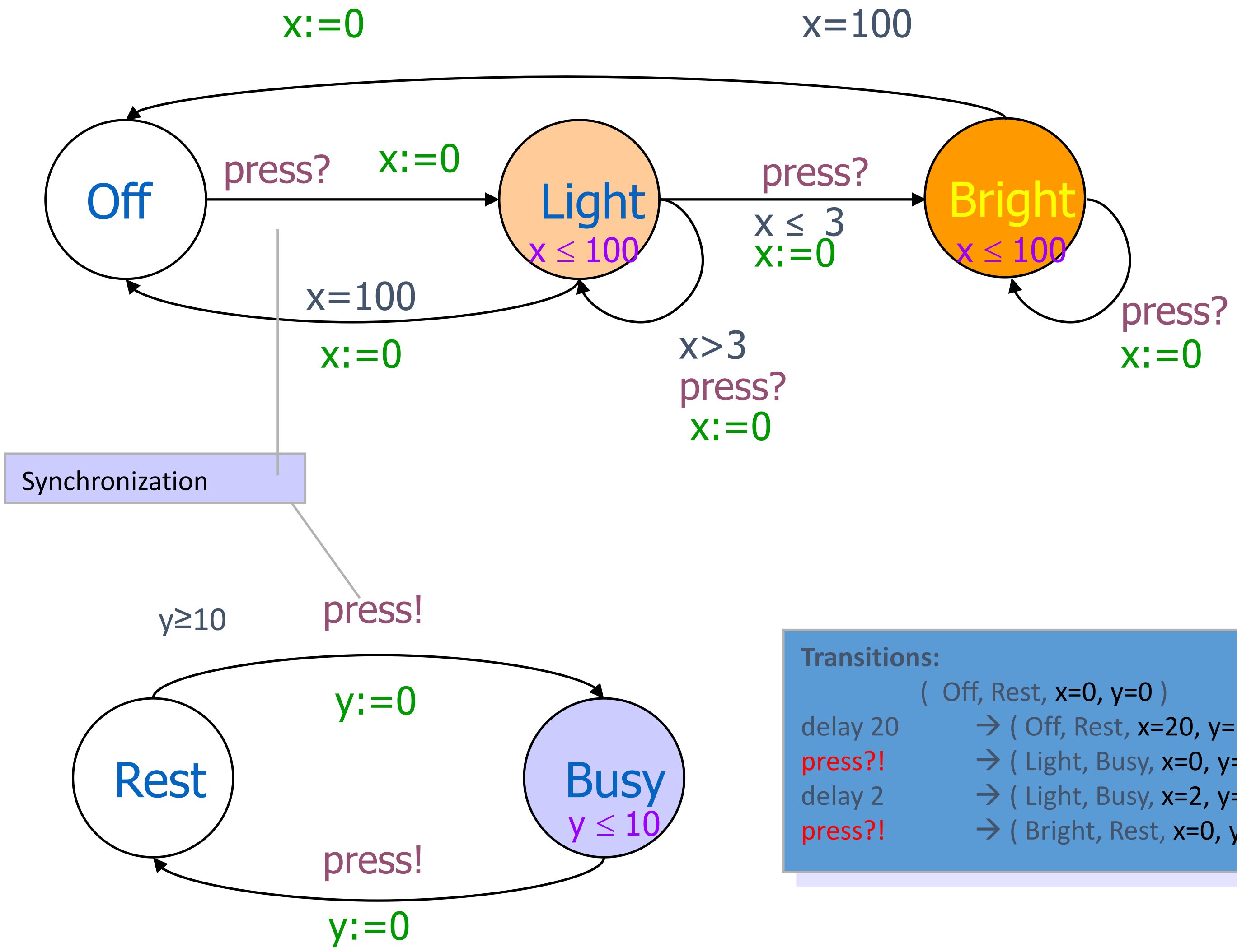
If requirements include **uncertainty**:

Automatically switch light off ***between*** 90-100 time units after switching on.



Location Invariant

Light Controller || User



Transitions:

(Off, Rest, x=0, y=0)

delay 20 → (Off, Rest, x=20, y=20)

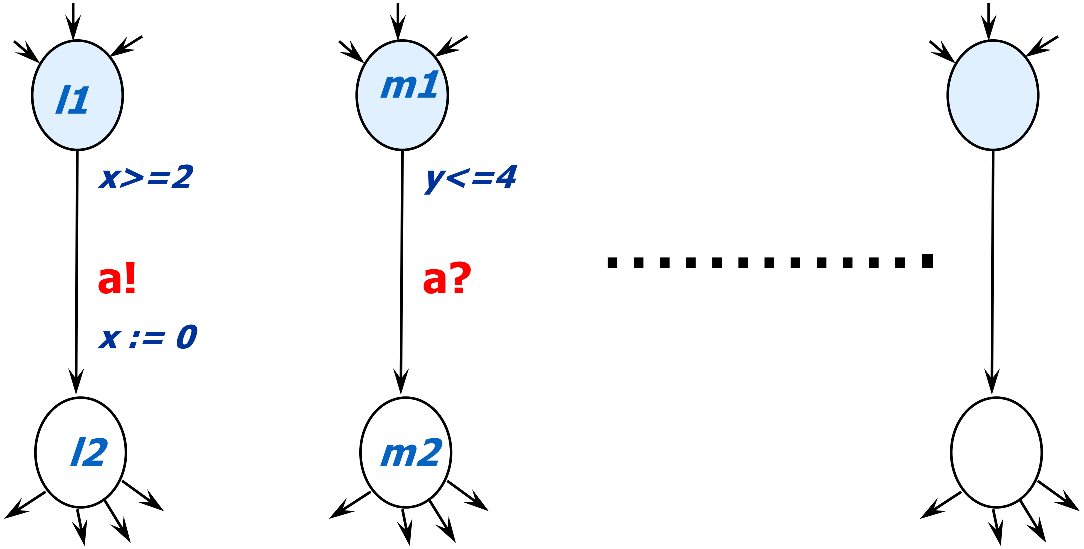
press?! → (Light, Busy, x=0, y=0)

delay 2 → (Light, Busy, x=2, y=2)

press?! → (Bright, Rest, x=0, y=0)

Networks of Timed Automata

(a'la CCS)



Two-way synchronization on *complementary* actions.

Closed Systems!

Example transitions

$(l1, m1, \ldots\ldots, x=2, y=3.5, \ldots\ldots) \xrightarrow{tau} (l2, m2, \ldots\ldots, x=0, y=3.5, \ldots\ldots)$

$\cancel{\xrightarrow{0.2}} (l1, m1, \ldots\ldots, x=2.2, y=3.7, \ldots\ldots)$

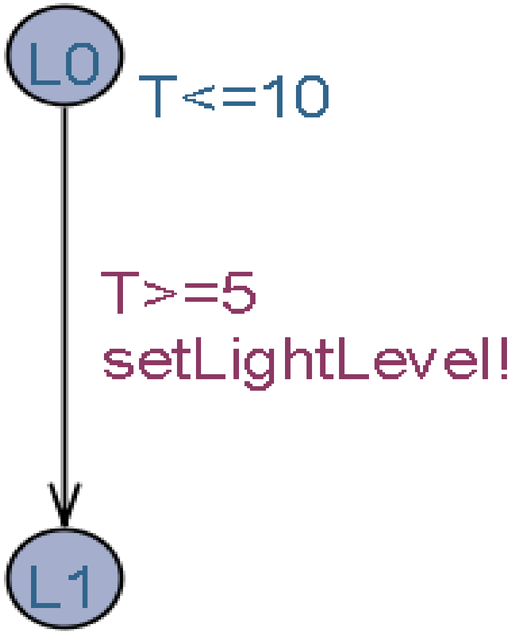
If **a** URGENT CHANNEL

How to model Timing Uncertainty?

- Unpredictable or variable timing
  - response time,
  - computation time
  - transmission time etc:

Example:
Light level must be adjusted
between 5 and 10 time units

Initially T=0

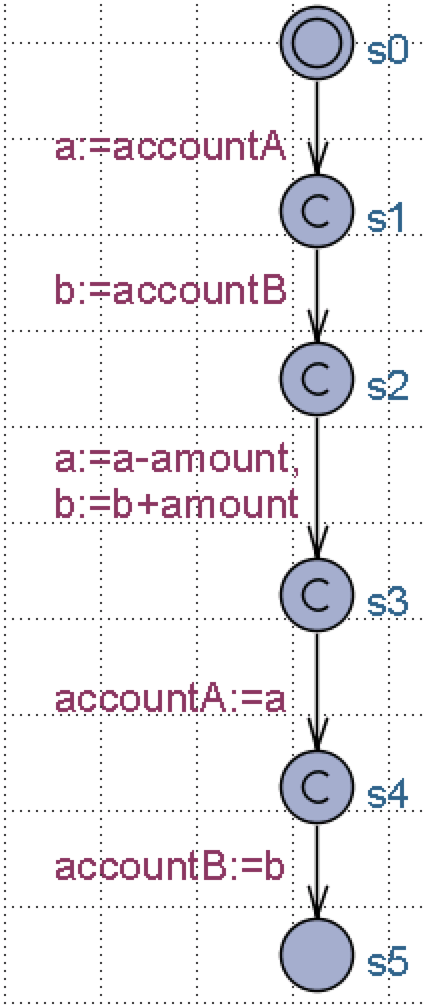


Comitted Locations

- Locations marked **C**
  - ***No delay*** in committed location.
  - No interleaving with parallel transitions
- Handy to model atomic sequences
- The use of committed locations reduces the number of states in a model, and allows for more space and time efficient analysis.

- Example:

Sequence s0 to s5 is executed atomically (no interleavings with concurrent actions)



Urgent Channels and Locations

- Locations marked **U**
  - ***No delay*** like in committed location.
  - But Interleaving permitted
- Channels declared “**`urgent chan`**”
  - Time doesn’t elapse when a synchronization is possible on a pair of urgent channels
  - Interleaving is allowed

Broadcast channel

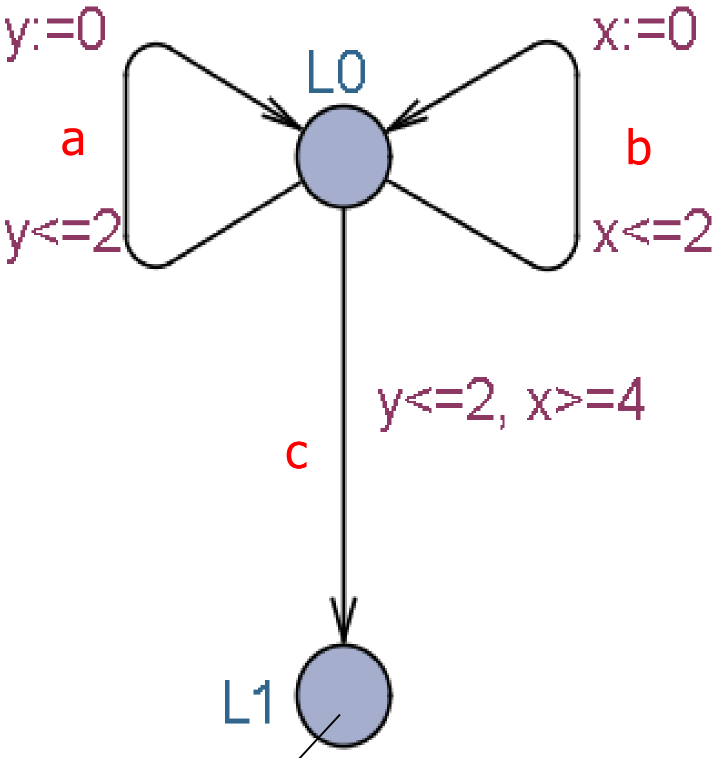
- Declaration: `broadcast chan  ch;`
- Sending process executes output action e.g. `ch!`
- Every automaton that listens to moves on synchronously

  ie. every automaton with enabled transition that is labeled with input action `ch?` moves on one step
- Provides non-blocking synchronization: even if zero listeners are enabled, sender process can progress anyway

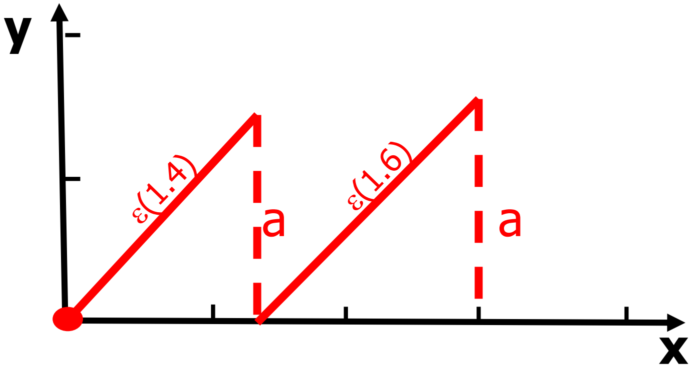
Other Uppaal features

- Bounded variable domains
  - `int [1..4] a;`
- C-like data-structures and user defined functions in declaration section
  - structs, arrays, and type definitions
    - `typedef int [0,n] t_id`
- non-deterministic assignment:
    - `select a:T        // select a random value from T`
- `forall`, `exists` in expressions
- Scalar sets (for giving unique ID’s)
- Process and channel **priorities**

Semantics: Timed traces



Reachable?



(L0, x=0,y=0)

$\rightarrow_{\varepsilon(1.4)}$

(L0, x=1.4,y=1.4)

$\rightarrow_{a}$

(L0, x=1.4,y=0)

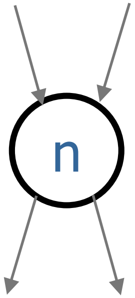
$\rightarrow_{\varepsilon(1.6)}$

(L0, x=3.0,y=1.6)

$\rightarrow_{a}$

(L0,x=3.0,y=0)

From explicit clock values to zones
(*from infinite to finite*)

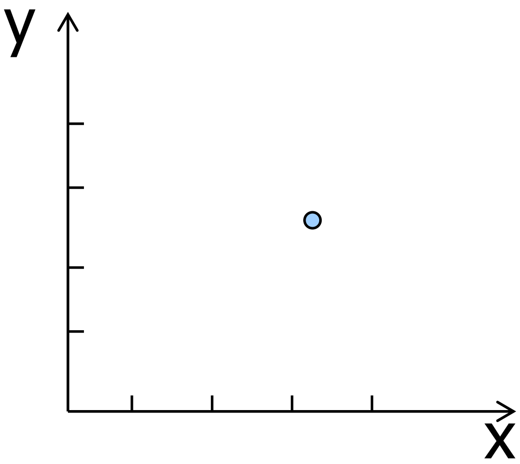


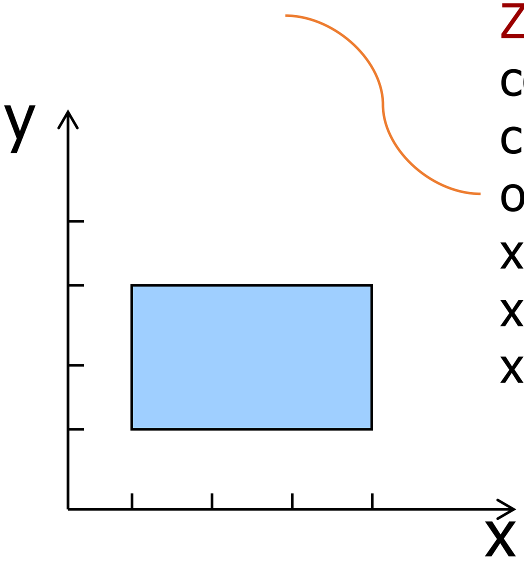
Explicit state

(n, x=3.2, y=2.5)

Symbolic state (set)

(n, 1 ≤ x ≤ 4, 1 ≤ y ≤ 3)

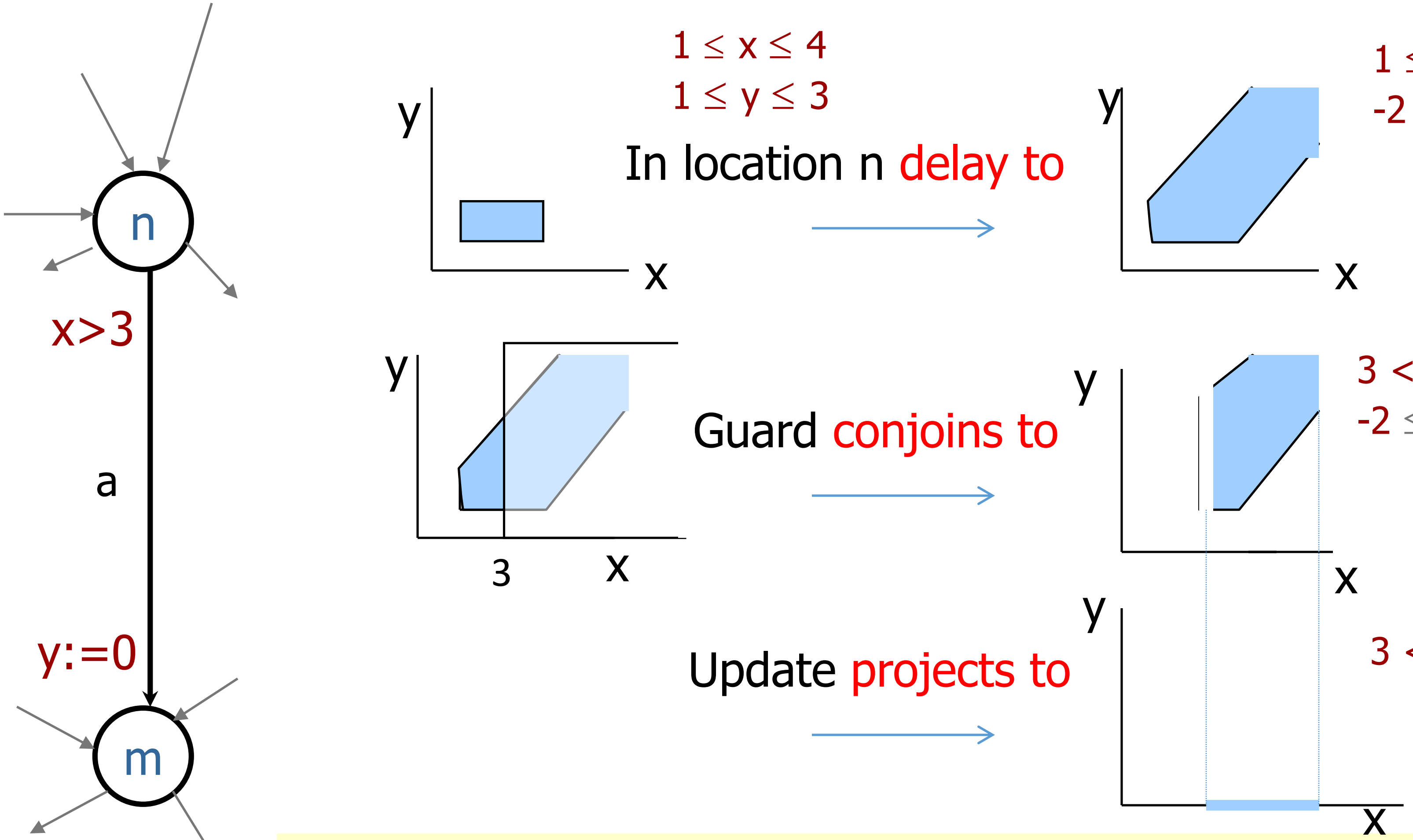




Zone:
conjunction of clock constraints of form:
x-y<=const1,
x<=const2,
x>=const3

Symbolic Transitions

Assume clock values when reaching location n



$1 \leq x \leq 4$
$1 \leq y \leq 3$

In location n delay to →

$1 \leq x$, $1 \leq y$
$-2 \leq x-y \leq 3$

Guard conjoins to →

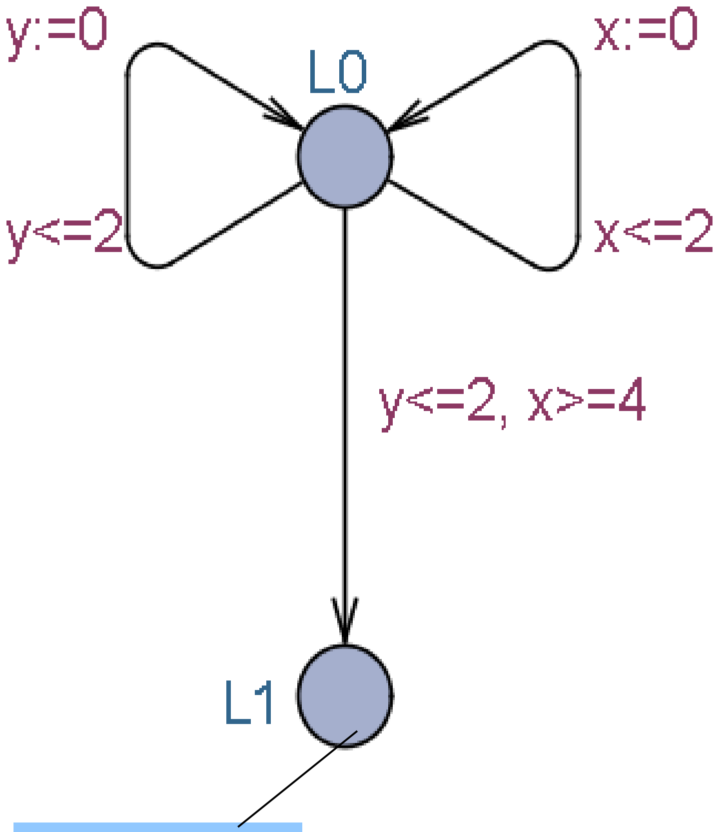
$3 < x$, $1 \leq y$
$-2 \leq x-y \leq 3$

Update projects to →

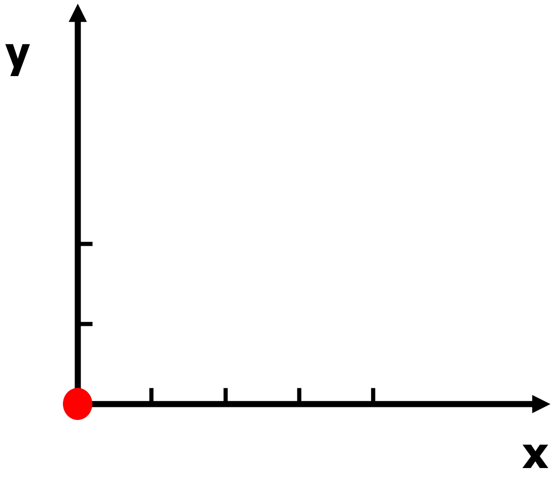
$3 < x$, $y=0$

Thus (n, $1 \leq x \leq 4$, $1 \leq y \leq 3$) $\rightarrow^{a}$ (m, $3 < x$, $y=0$)

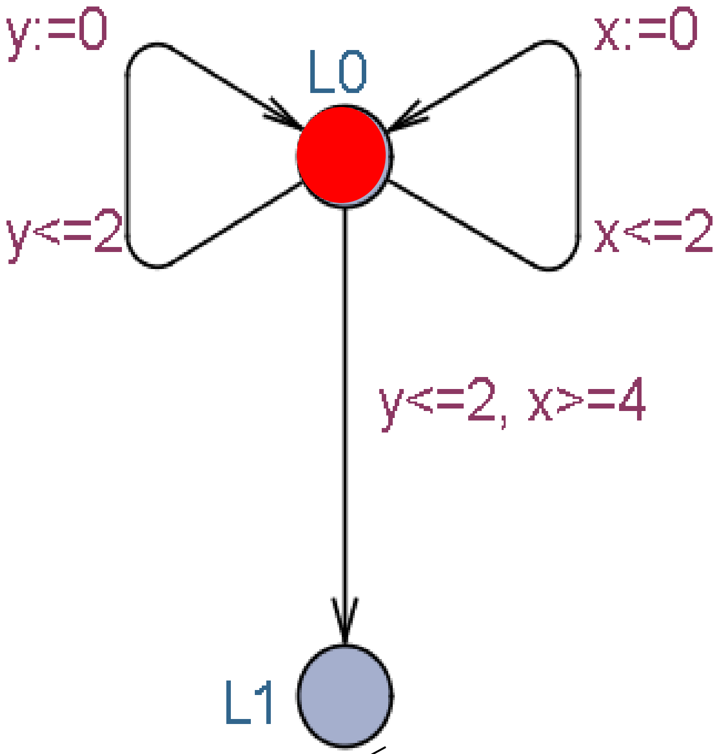
Symbolic state exploration



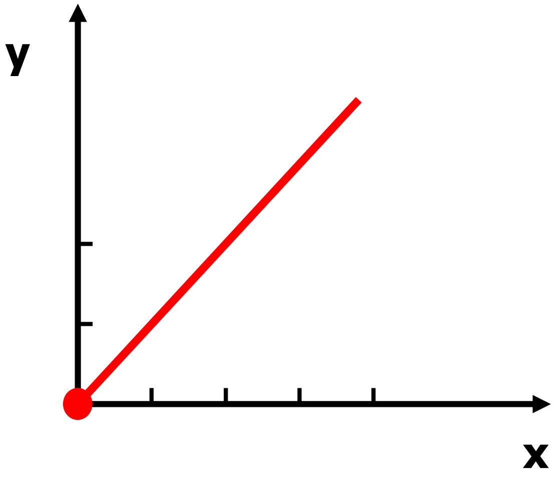
Reachable?



Symbolic Exploration

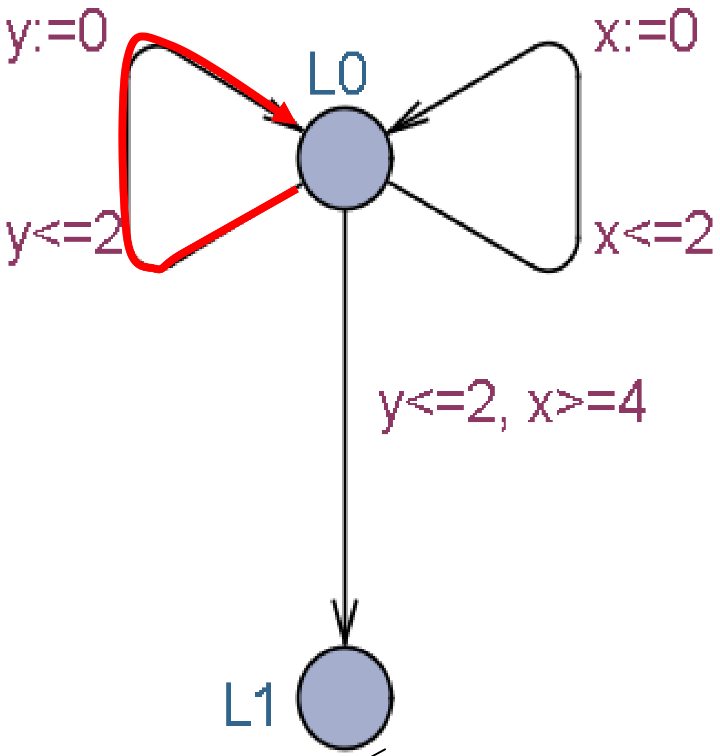


Reachable?

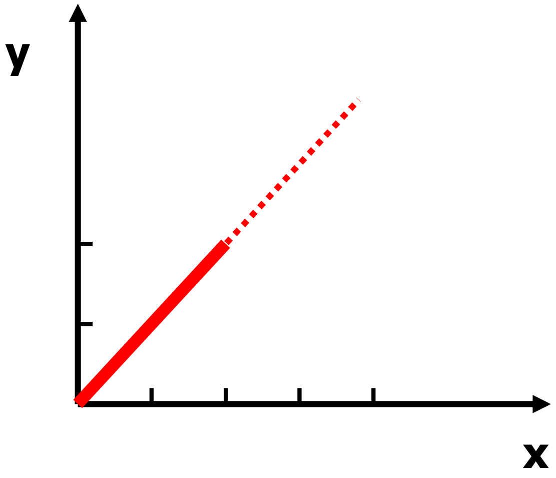


Delay

Symbolic Exploration

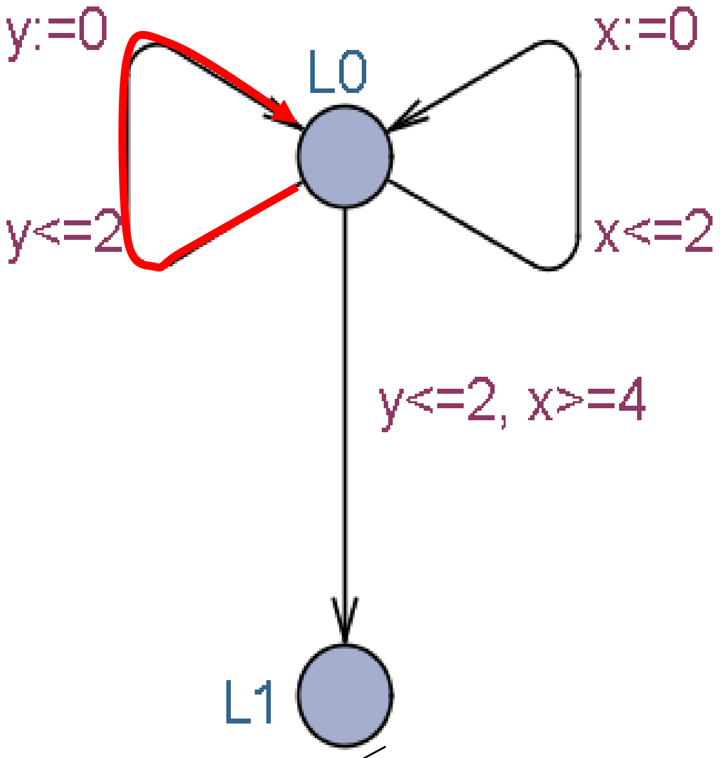


Reachable?



Left

Symbolic Exploration



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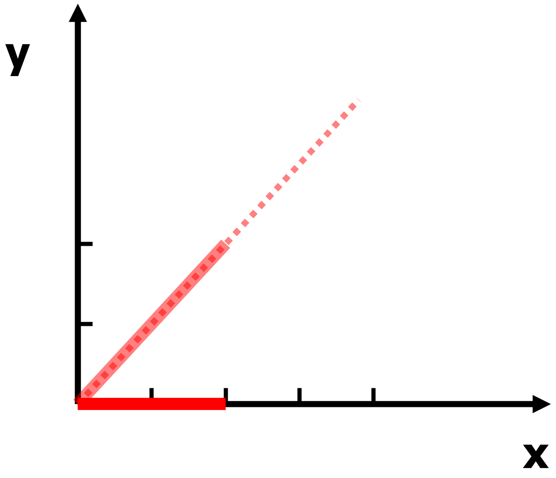
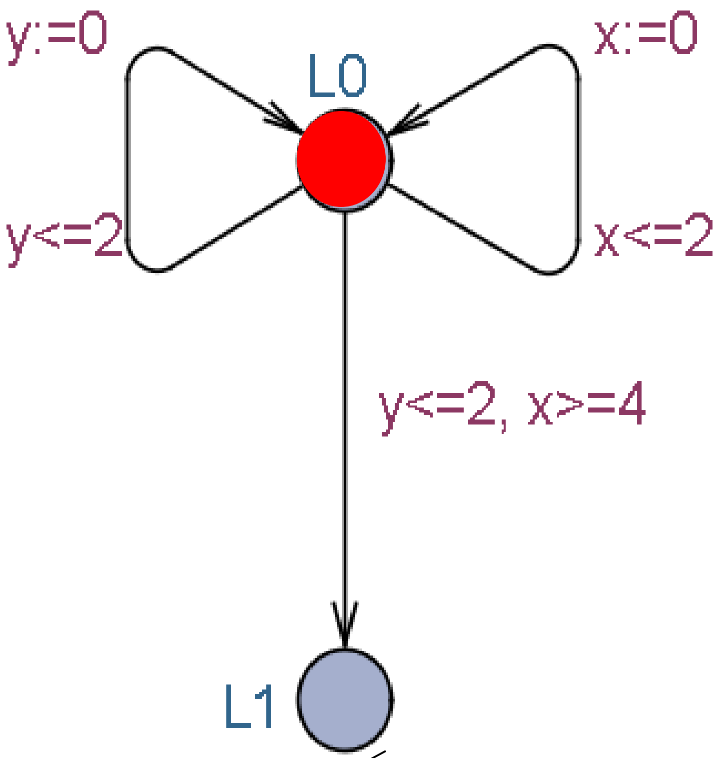
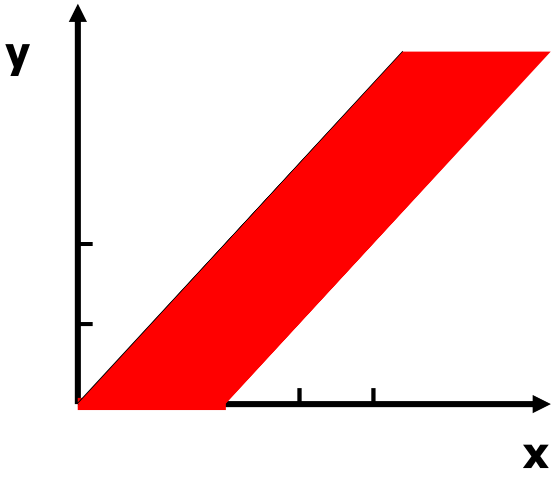


Left

Symbolic Exploration

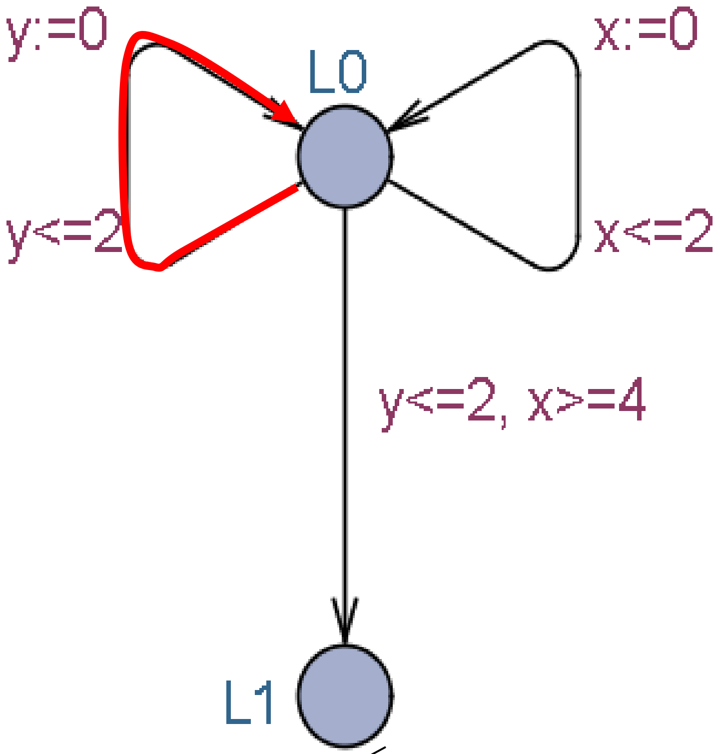


Reachable?

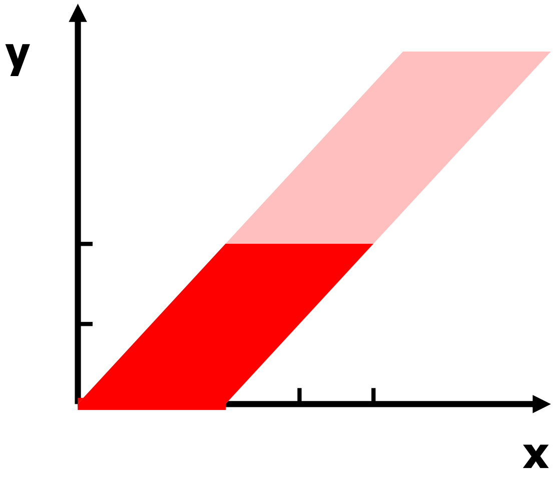


Delay

Symbolic Exploration



Reachable?



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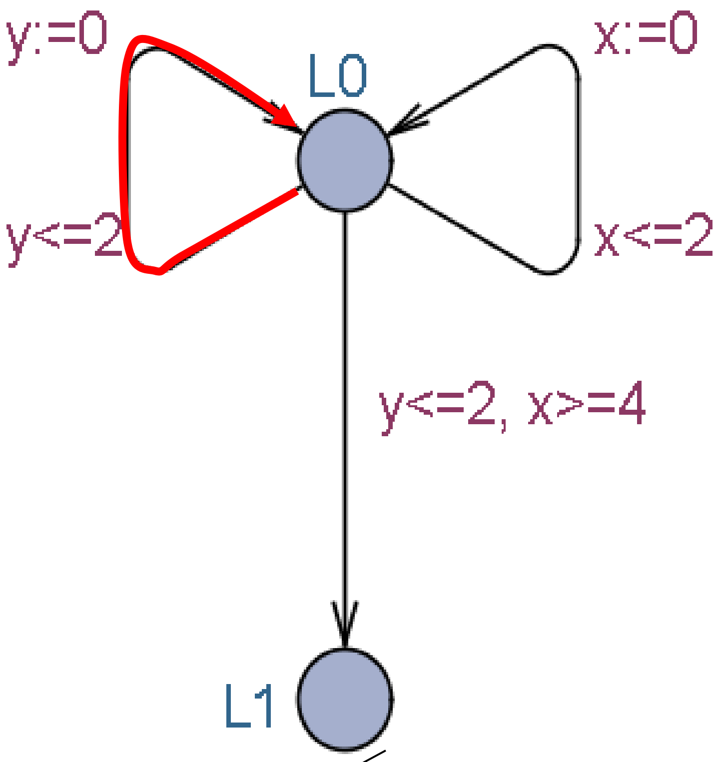
Symbolic Exploration



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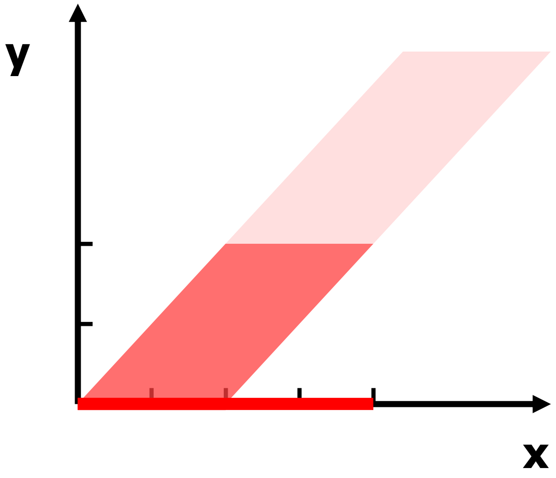
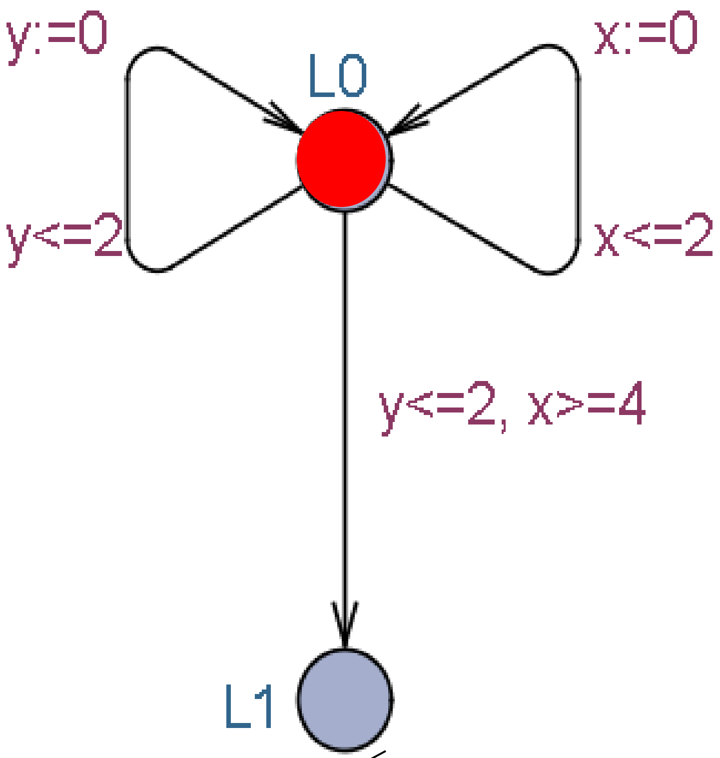


Left

Symbolic Exploration



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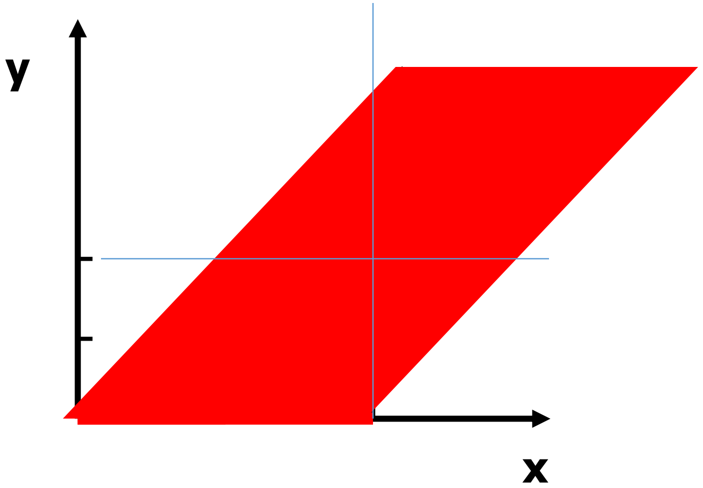
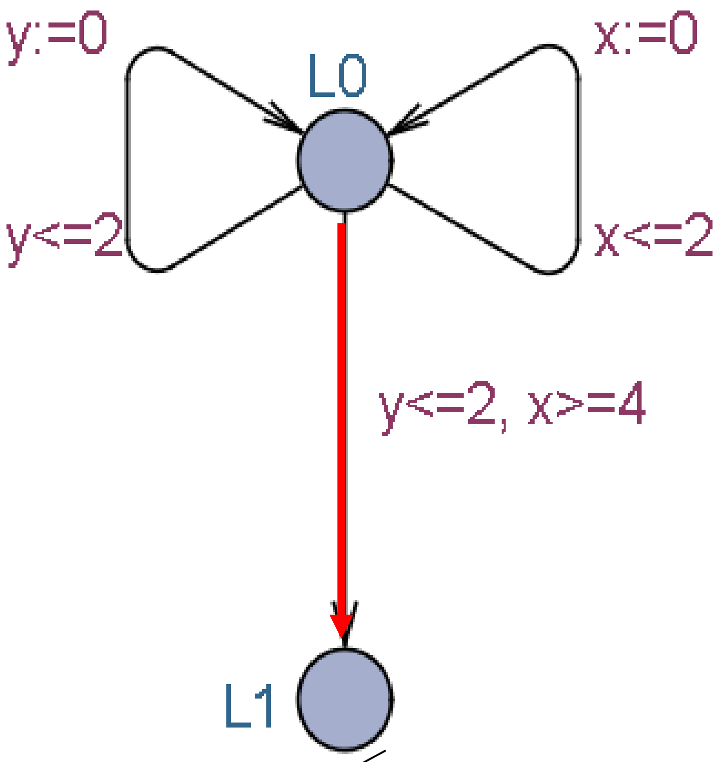
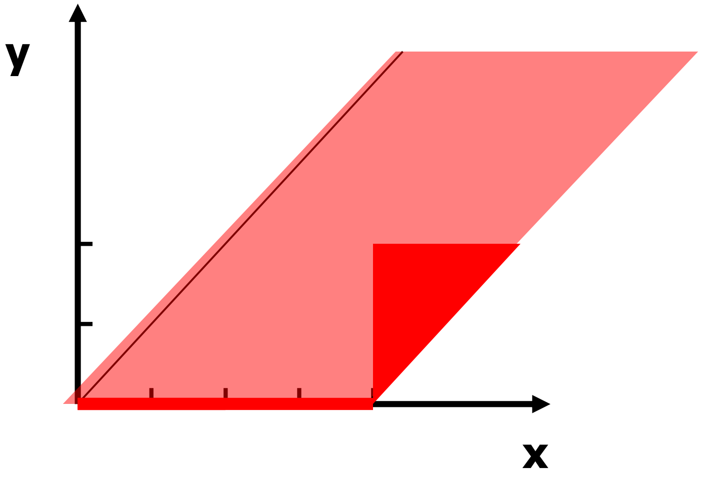


Delay

Symbolic Exploration



Reachable?

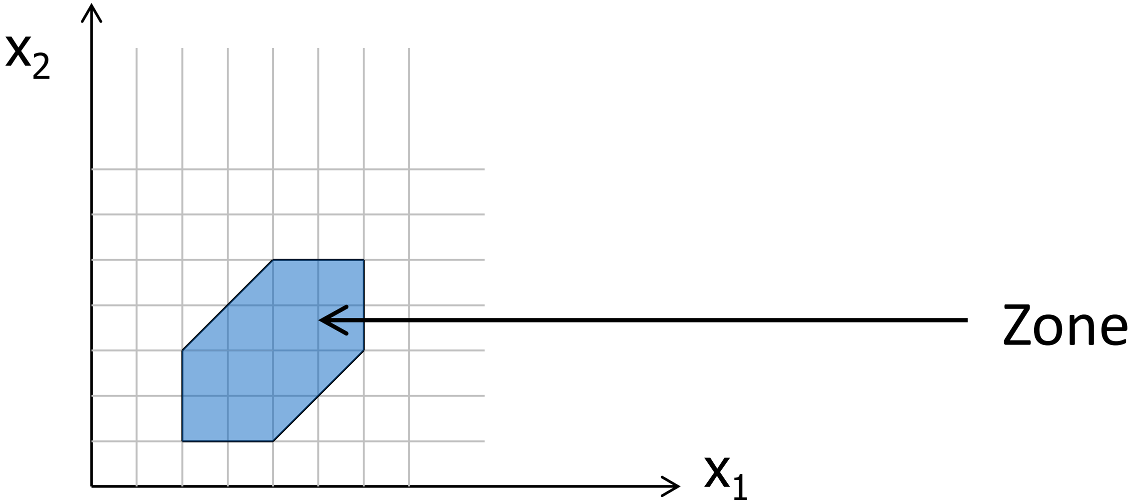


Down

Difference Bound Matrix

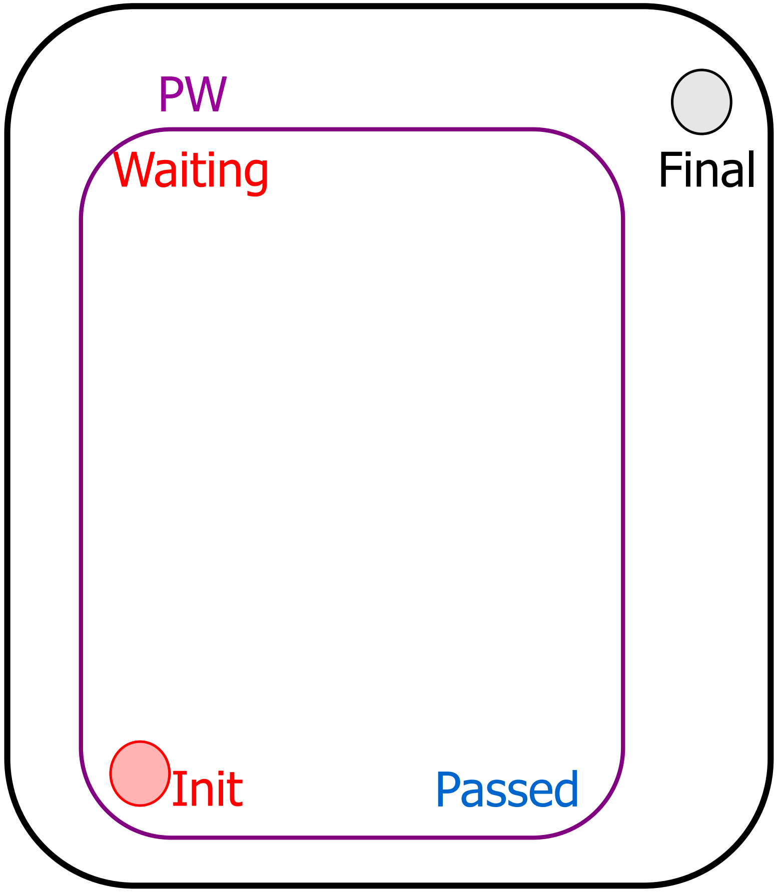
$x_0-x_0<=0$	$x_0-x_1<=-2$	$x_0-x_2<=-1$
$x_1-x_0<=6$	$x_1-x_1<=0$	$x_1-x_2<=3$
$x_2-x_0<=5$	$x_2-x_1<=1$	$x_2-x_2<=0$

$$x_i-x_j<=c_{ij}$$



Forward Reachability Algorithm

Init -> Final ?



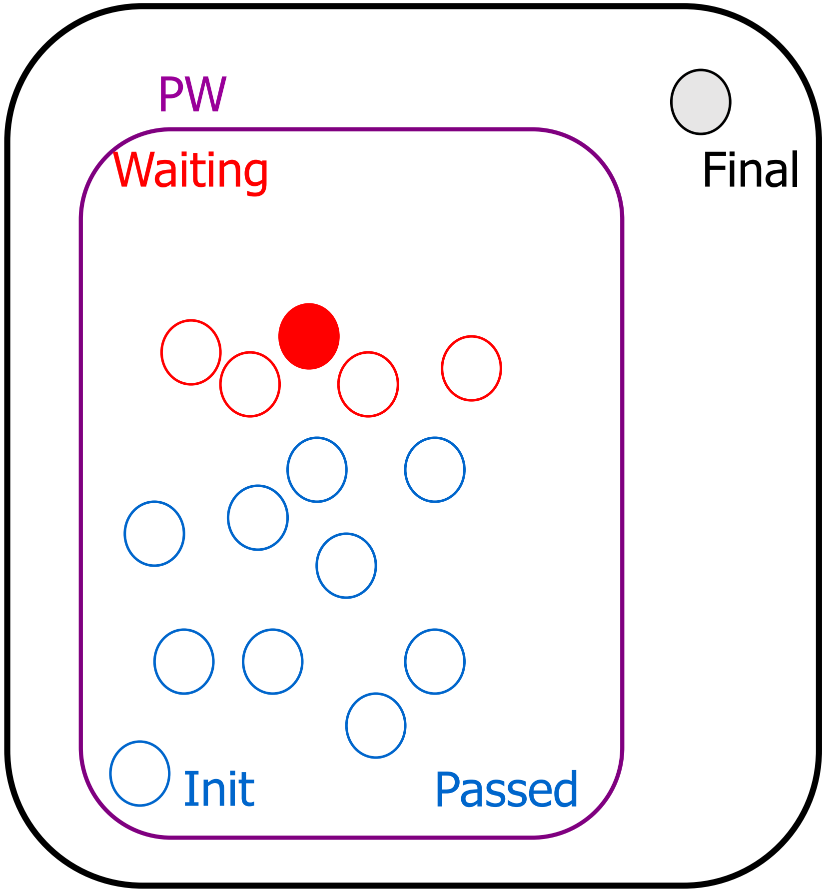
```
INITIAL  Passed := Ø;
         Waiting := {($n_0$,$Z_0$)}

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all $(n,Z) \rightarrow (n',Z')$:
      if for some (n',Z'') in Passed $Z' \subseteq Z''$
      then continue
      else add (n',Z') to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

Init -> Final ?



```
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         Waiting := {($n_0$,$Z_0$)}

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all (n,Z)→(n′,Z′):
      if for some (n′,Z″) $Z' \subseteq Z''$ continue
      else add (n′,Z′) to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```
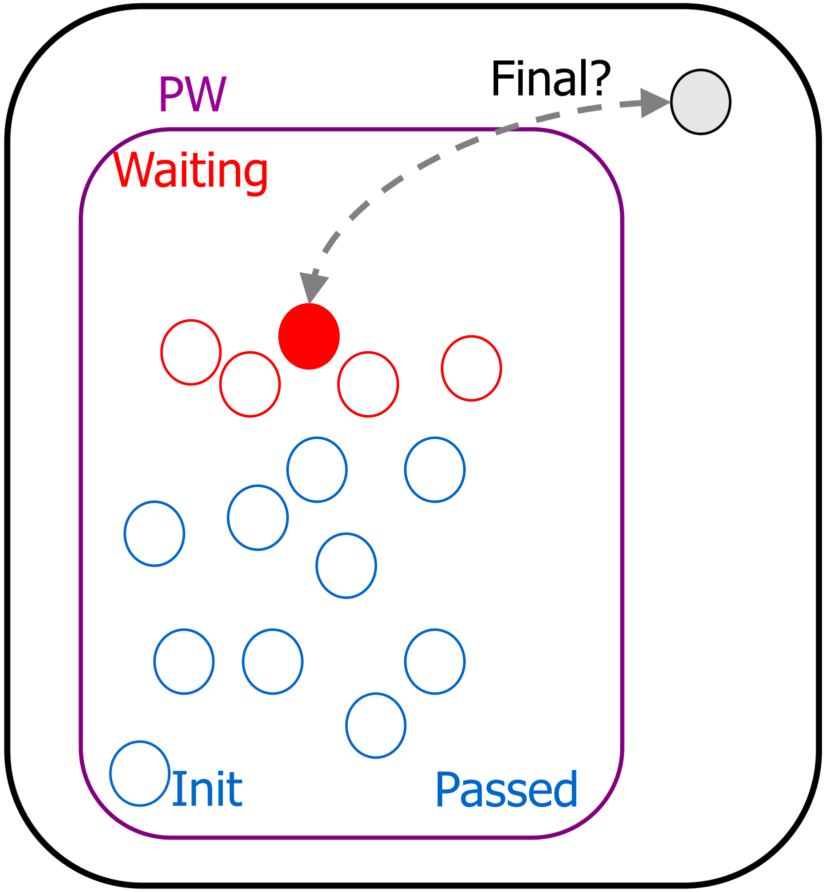

Forward Reachability Algorithm

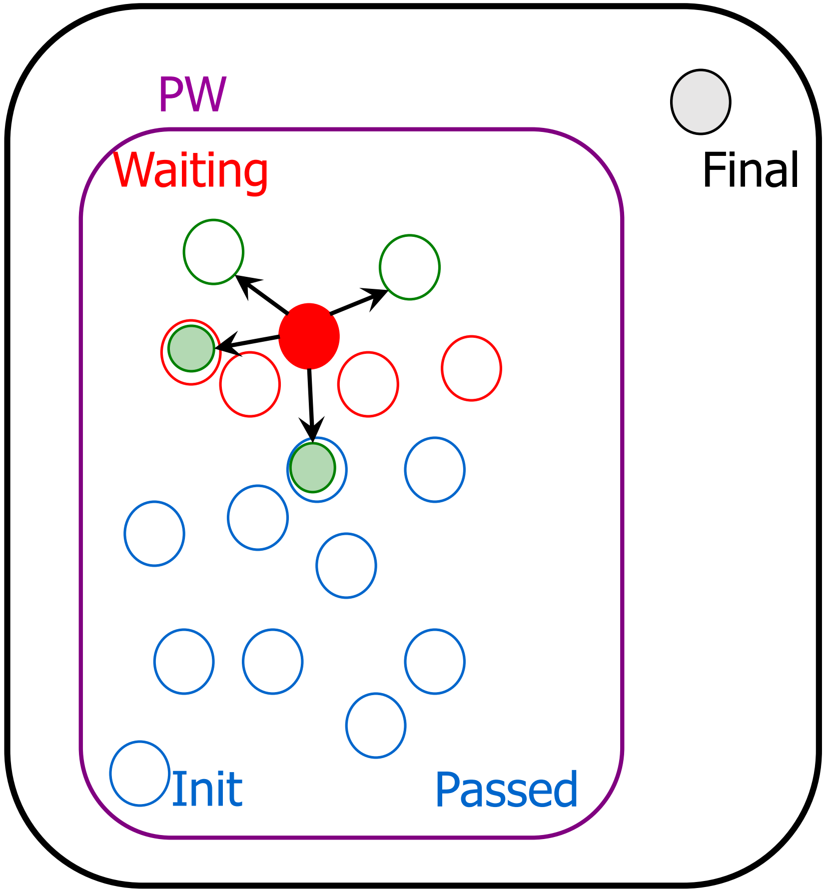
Init -> Final ?



```
INITIAL  Passed := $\emptyset$;
         Waiting := $\{(n_0,Z_0)\}$

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all $(n,Z)\rightarrow(n',Z')$:
      if for some $(n',Z'')$ $Z' \subseteq Z''$ continue
      else add $(n',Z')$ to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = $\emptyset$
return false
```

Forward Reachability Algorithm

Init -> Final ?



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REPEAT
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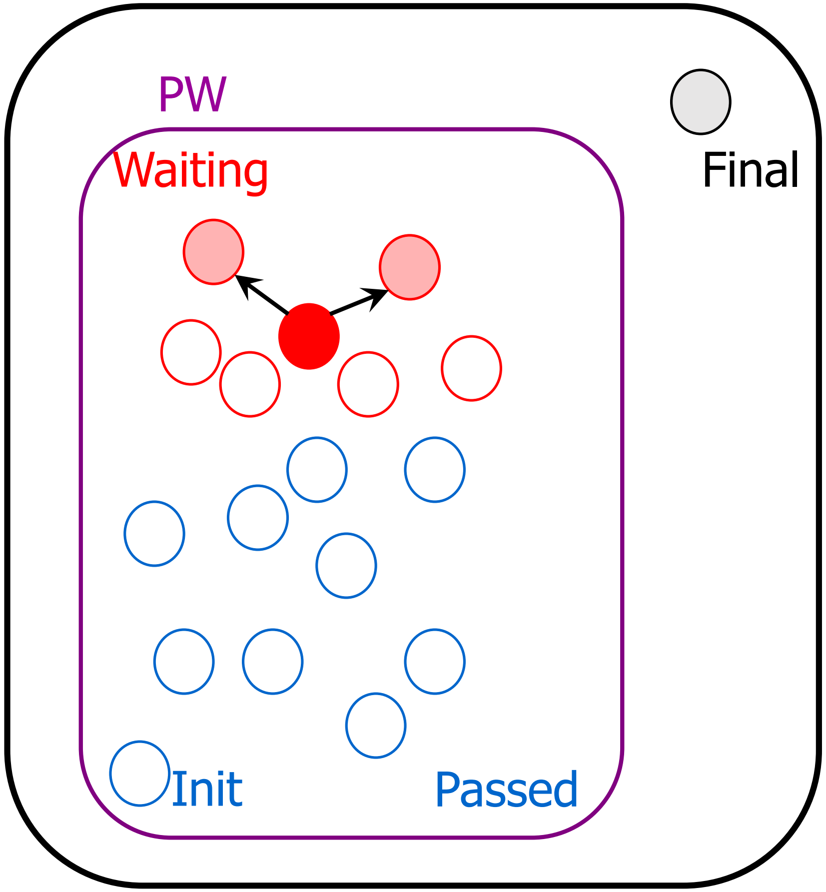
   if (n,Z) = Final return true
   for all (n,Z)→(n',Z'):
      if for some (n',Z'') Z'⊆ Z'' continue
      else add (n',Z') to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

Init -> Final ?



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REPEAT
    pick (n,Z) in Waiting
    if (n,Z) = Final return true
    for all (n,Z)→(n',Z'):
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        else add (n',Z') to Waiting
        move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```
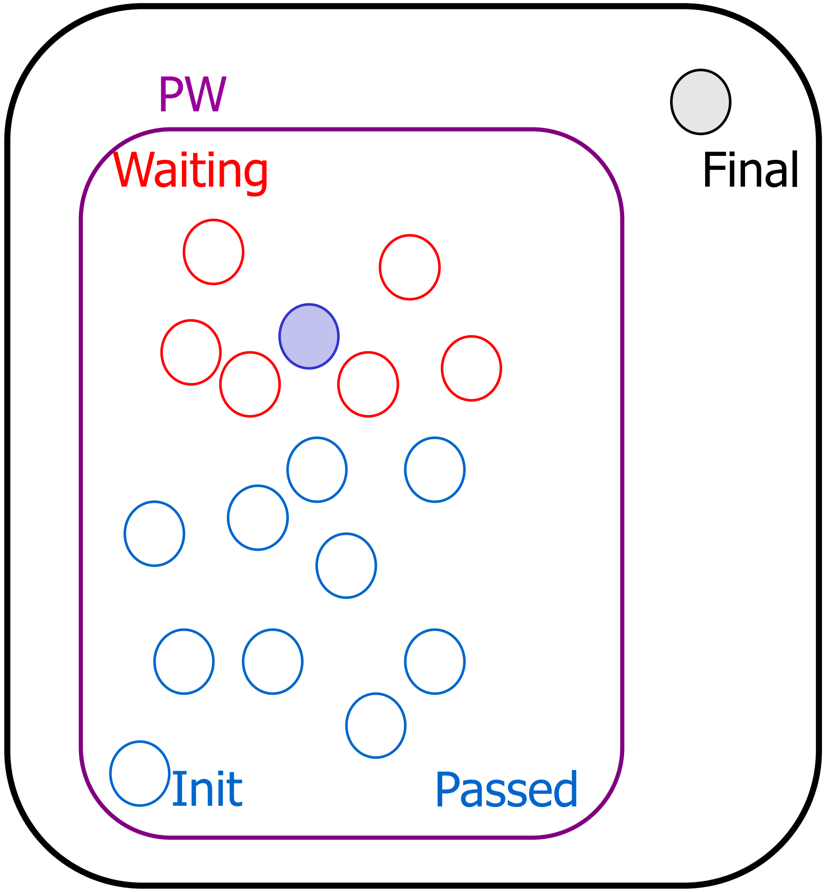

Forward Reachability Algorithm

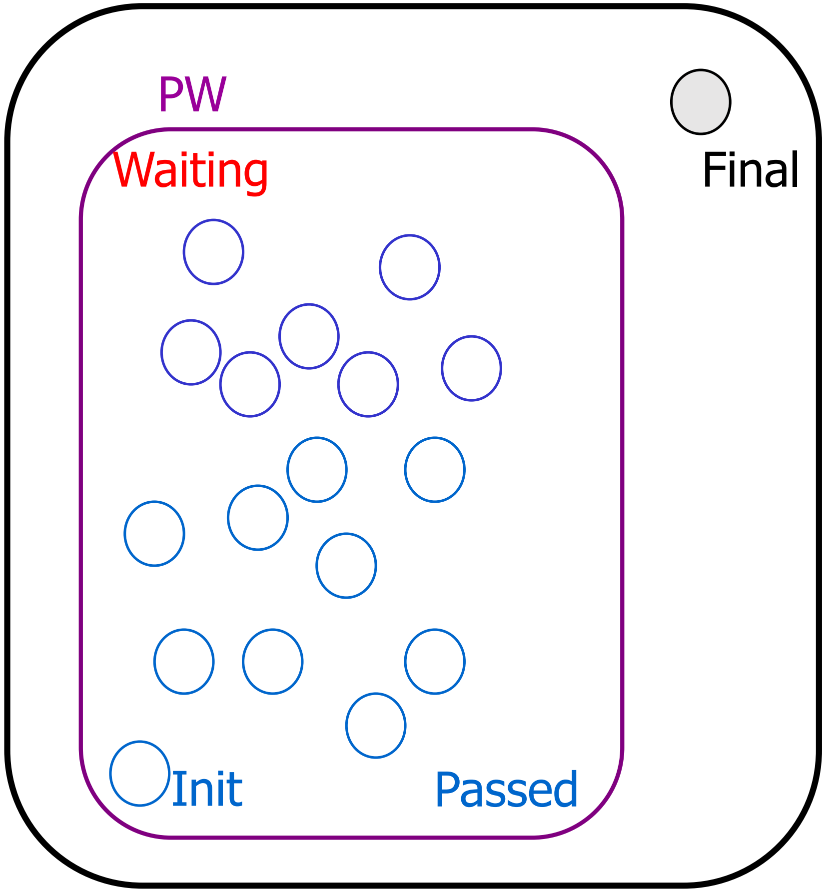
Init -> Final ?



```
INITIAL  Passed := Ø;
             Waiting := {($n_0$,$Z_0$)}

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all (n,Z)→(n′,Z′):
      if for some (n′,Z′) Z′⊆ Z″ continue
      else add (n′,Z′) to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

Init -> Final ?



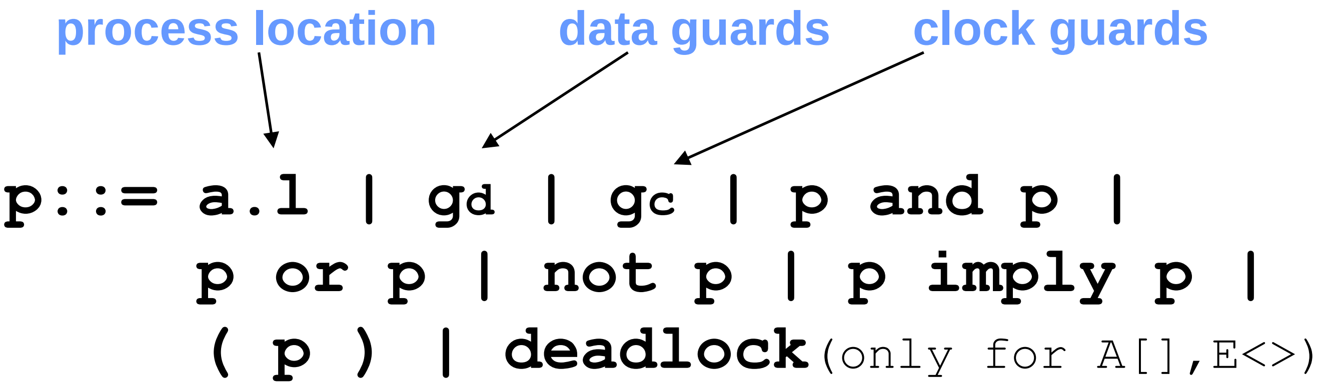
```
INITIAL  Passed := Ø;
              Waiting := {$(n_0,Z_0)$}

REPEAT
    pick (n,Z) in Waiting
    if (n,Z) = Final return true
    for all (n,Z)→(n',Z'):
        if for some (n',Z'') $Z' \subseteq Z''$ continue
        else add (n',Z') to Waiting
        move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Specification (Query) Language
TCTL

UPPAAL Property Specification Language

- **A[] p** *always*
- **A<> p** *inevitable*

- **E<> p** *Possible*
- **E[] p** *potentially always*
- **P --> q** *leads-to*



process location

data guards

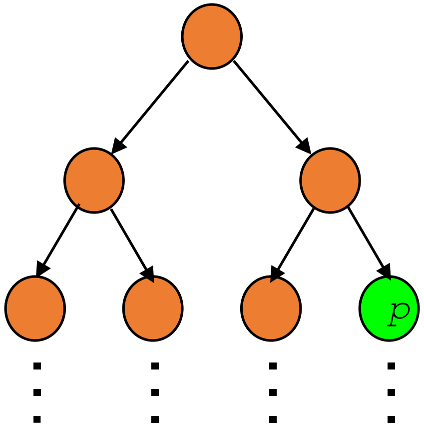
clock guards

p::= a.l | g_d | g_c | p and p |
p or p | not p | p imply p |
(p) | deadlock (only for A[],E<>)

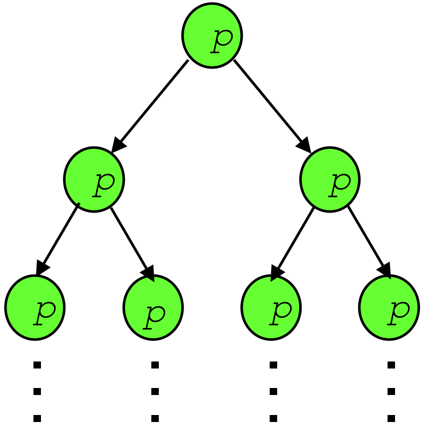
Example:
`A[] (mc1.finished and mc2.finished) imply (accountA+accountB==200)`

Uppaal "Computation Tree Logic"

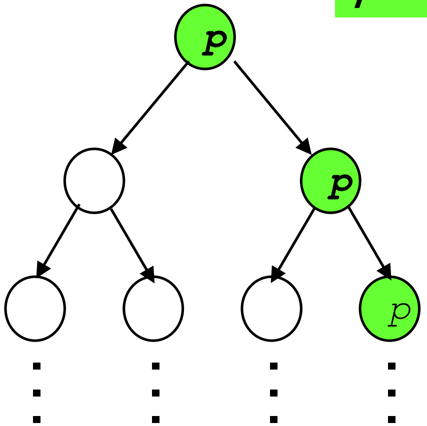
`E<> p` *Possible*



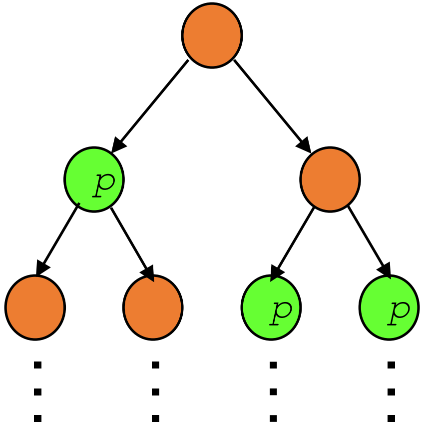
`A[] p` *always*



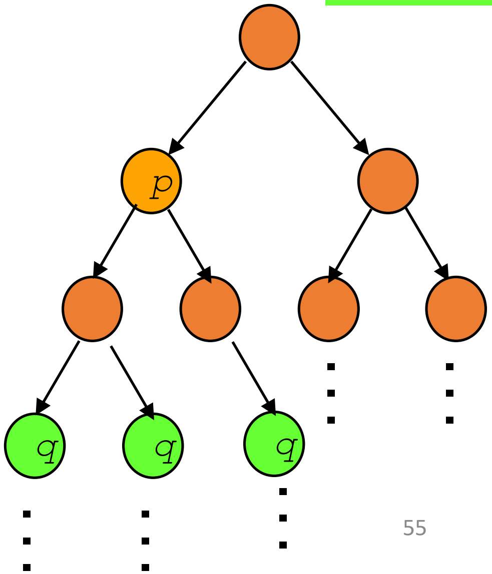
`E[] p` *potentially always*



`A<> p` *inevitable*



`p --> q` *leads-to*



Logical Specifications

- Validation Properties
  - Possibly: E<> p
- Safety Properties
  - Invariant: A[] p
  - Possibly Inv.: E[] P
- Liveness Properties
  - Eventually: A<> p
  - Leads_to: p --> p
- Bounded Liveness
  - Leads to within: p --> $_{\leq t}$ q

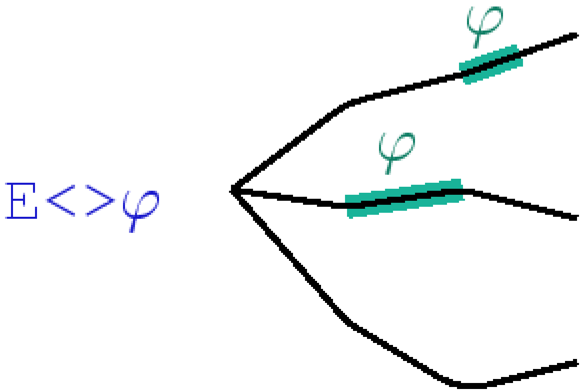
The expressions p and q

- must be type safe, side effect free, and evaluate to a boolean.

- only references to integer variables, constants, clocks, and locations are allowed (and arrays of these).

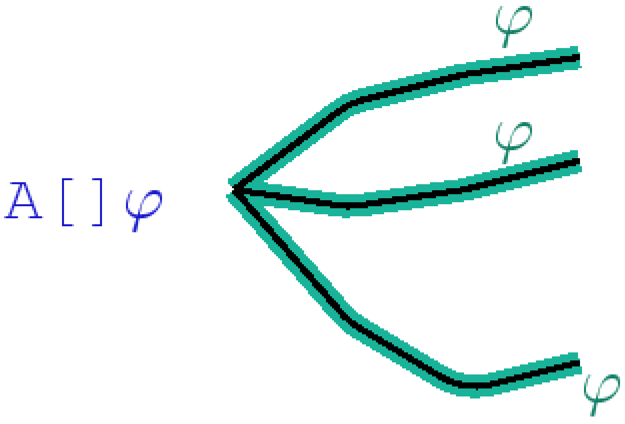
Logical Specifications

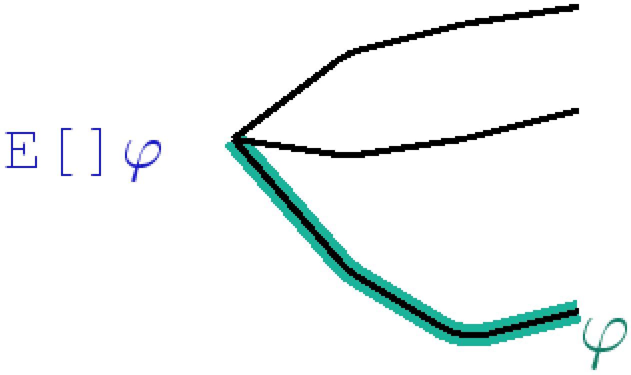
- Validation Properties
  - Possibly: E<> φ
- Safety Properties
  - Invariant: A[] φ
  - Pos. Inv.: E[] φ
- Liveness Properties
  - Eventually: A<> φ
  - Leadsto: φ --> ψ
- Bounded Liveness
  - Leads to within: φ --> $_{\leq t}$ ψ



Logical Specifications

- Validation Properties
  - Possibly: E<> *p*
- Safety Properties
  - Invariant: A[] φ
  - Pos. Inv.: E[] φ
- Liveness Properties
  - Eventually: A<> φ
  - Leadsto: φ --> ψ
- Bounded Liveness
  - Leads to within: φ --> $_{\leq t}$ ψ





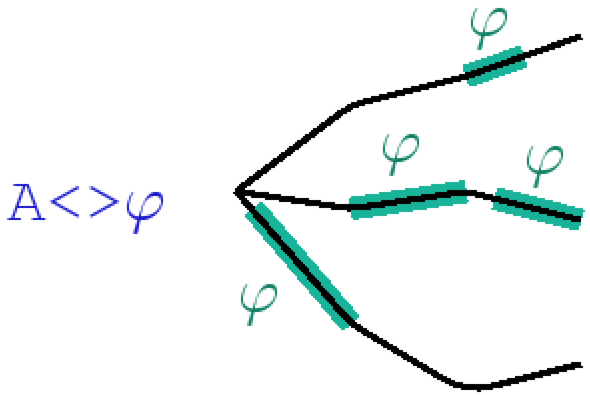
Logical Specifications

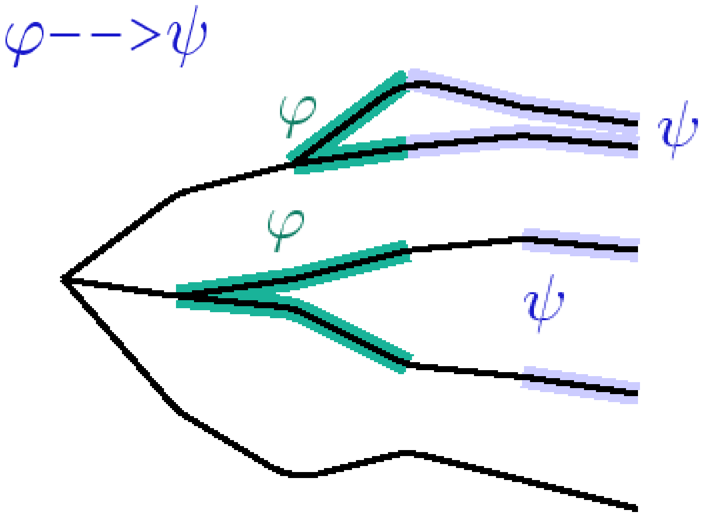
- Validation Properties
  - Possibly: E<> φ

- Safety Properties
  - Invariant: A[] φ
  - Pos. Inv.: E[] φ

- Liveness Properties
  - Eventually: A<> φ
  - Leads to: φ --> ψ

- Bounded Liveness
  - Leads to within: φ --> $_{\leq t}$ ψ





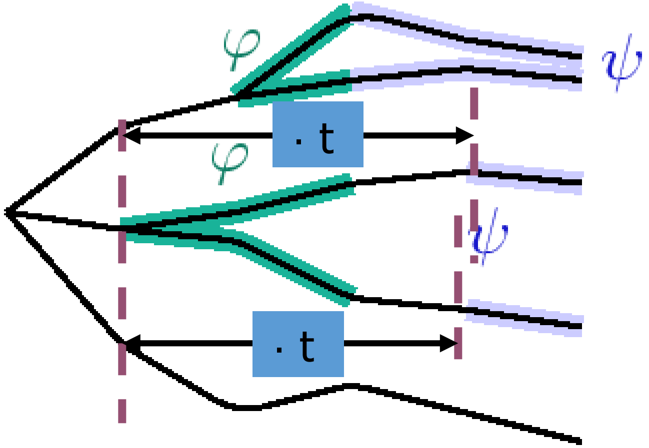
Logical Specifications

- Validation Properties
  - Possibly: E<> φ

- Safety Properties
  - Invariant: A[] φ
  - Pos. Inv.: E[] φ

- Liveness Properties
  - Eventually: A<> φ
  - Leadsto: φ --> ψ

- Bounded Liveness
  - Leads to within: φ --> $_{\leq t}$ ψ



Jug Example

- Safety: Never overflow.
  - A[] forall(i:id_t) level[i] <= capa[i]

- Validation/Reachability: How to get 1 unit.
  - E<> exists(i:id_t) level[i] == 1

Train-Gate Crossing Example

- Safety: One train on crossing at a time.
  - `A[] forall (i : id_t) forall (j : id_t)`
    `Train(i).Cross && Train(j).Cross imply i == j`
- Liveness: Approaching trains eventually arrive on crossing.
  - `Train(0).Appr --> Train(0).Cross`
  - `Train(1).Appr --> Train(1).Cross`
  - …
- No deadlock.
  - `A[] not deadlock`