Definition 1 (Binary Relation)**.** A binary relation R between sets A and B is the set

$$R \subseteq A \times B : \forall x \in A, \forall y \in B : xRy \iff (x, y) \in R \ .$$

Definition 2 (Domain of a Binary Relation)**.** The **domain** of $R \subseteq A \times B$ is the set

$$Dom(R) = \{x \in A : \exists y \in B : xRy\} \ .$$

Definition 3 (Image of a Binary Relation)**.** The **image of** A **under** $R \subseteq A \times B$ is the set

$$Im(R) = \{y \in B : \exists x \in A : xRy\} \ .$$

Definition 4 (Field of a Binary Relation)**.** The **field of** R is the set

$$Field(R) = Dom(R) \cup Im(R) \ .$$

Definition 5 (Injective Binary Relation)**.** A binary relation $R \subseteq A \times B$ is **injective** (or **left-unique**) if

$$\forall x, z \in A, \forall y \in B : xRy \wedge zRy \implies x = z \ .$$

Definition 6 (Functional Binary Relation)**.** A binary relation $R \subseteq A \times B$ is **functional** (or **right-unique**) if

$$\forall x \in A, \forall y, z \in B : xRy \wedge xRz \implies y = z \ .$$

Definition 7 (One-to-one Binary Relation)**.** A binary relation R is **one-to-one** if it is **injective** and **functional**.

Definition 8 (Left-total Binary Relation)**.** A binary relation $R \subseteq A \times B$ is **left-total** if

$$\forall x \in A \ \exists y \in B : xRy \ .$$

Definition 9 (Surjective Binary Relation)**.** A binary relation $R \subseteq A \times B$ is **surjective** (or **right-total**, or **onto**) if

$$\forall y \in B \ \exists x \in A : xRy \ .$$

Definition 10 (Mapping)**.** A binary relation is a **mapping** (or a **function**) $f : A \to B$ if it is functional (right-unique) and left-total.

Definition 11 (Mapping Domain)**.** The **domain** of a mapping $f : A \to B$ is the set A.

$$Dom(f) = A \ .$$

Definition 12 (Mapping Range)**.** The **range** of a mapping $f : A \to B$ is the set B.

Definition 13 (Mapping Image)**.** The **image** of a mapping $f : A \to B$ is the set

$$f(A) = \{f(a) : a \in A\} \subseteq B \ .$$

Definition 14 (Injective Mapping)**.** An **injection** is an injective mapping – a binary relation that is left-unique, right-unique, and left-total

Definition 15 (Surjective Mapping)**.** A **surjection** (or **onto mapping**) is a surjective mapping – a binary relation that is right-unique, left-total, and right-total.

Definition 16 (Bijective Mapping)**.** A mapping is a **bijection** (or **one-to-one correspondence**) is a mapping which is injective and surjective.

Definition 17 (Cardinality)**. Cardinality** is a measure of the number of elements in a given set.

$|A| = |B|$ if there exists a bijection $A \to B$.

$|A| \leqslant |B|$ if there exists an injection $A \to B$.

$|A| < |B|$ if there exists an injection $A \to B$, but no bijection $A \to B$.

Definition 18 (Infinite Set)**.** A set A is **infinite** if there exists $A' \subset A$ such that $|A'| = |A|$.

Definition 19 (Finite Countable Set)**.** A finite set A is **countable** if there exists an injection $A \to \mathbb{N}$.

Definition 20 (Countably Infinite Set)**.** A set A is **countably infinite** if there exists a bijection $A \to \mathbb{N}$.