Exercise 1. Show that $(\mathbb{R}, +) \cong (\mathbb{R}^+, \cdot)$.

Exercise 2. Show that $\mathbb{Z} \cong \mathbb{Q} \setminus \{0\}$ consisting of the elements in the form 2^n.

Exercise 3. Prove that $\mathbb{Z} \cong n\mathbb{Z}$ for $n \neq 0$.

Exercise 4. Prove that $\mathbb{C}*$ is isomorphic to the subgroup of $GL_2(\mathbb{R})$ consisting of matrices of the form

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} .$$

Exercise 5. Prove or disprove: $U(8) \cong \mathbb{Z}_4$.

Exercise 6. Prove that $U(8)$ is isomorphic to a group of matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} , \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} .$$

Exercise 7. Show that any cyclic group of order n is isomorphic to $\mathbb{Z}_n$.

Exercise 8. Prove that $\mathbb{Q}$ is not isomorphic to $\mathbb{Z}$.

Exercise 9. Let $G = \mathbb{R} \setminus \{-1\}$ and define a binary operation on G by

$$a \circ b = a + b + ab \ .$$

Prove that G is a group under this operation. Show that $(G, \circ)$ is isomorphic to the multiplicative group of nonzero real numbers.

Exercise 10. Prove that the subgroup of $\mathbb{Q} \setminus \{0\}$ consisting of elements of the form $2^m 3^n$ for $m, n \in \mathbb{Z}$ is an internal direct product isomorphic to $\mathbb{Z} \times \mathbb{Z}$.