# 1 Theory

Indices of letters:

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |

Measure of Roughness (**MR**) is a measure how much a distribution differs from a uniform distribution.

$$\mathbf{MR} = \sum_i \left(p_i - \frac{1}{26}\right)^2 = \sum_i p_i^2 - 2\frac{1}{26}\underbrace{\sum_i p_i}_{=1} + \underbrace{\sum_i \left(\frac{1}{26}\right)^2}_{=26\cdot\frac{1}{26^2}} = \sum_i p_i^2 - \frac{1}{26} \approx \sum_i p_i^2 - 0.038 \ .$$

$\sum_i p_i^2$ os the probability that any two letters randomly selected from a distribution will be the same. Index of coincidence **IC** is an approximation to $\sum_i p_i^2$. In a set of $N$ elements, element $a$ can form $\binom{f_a}{2} = \frac{f_a\cdot(f_a-1)}{2}$ pairs, where $f_a$ is the number of times letter $a$ appears in the set. The total number of possible pairs in a set of $N$ letters is $\binom{N}{2} = \frac{N\cdot(N-1)}{2}$. The probability that two randomly selected letters will be "A"-s is

$$\frac{\binom{f_A}{2}}{\binom{N}{2}} = \frac{f_a\cdot(f_a-1)}{N\cdot(N-1)} \ ,$$

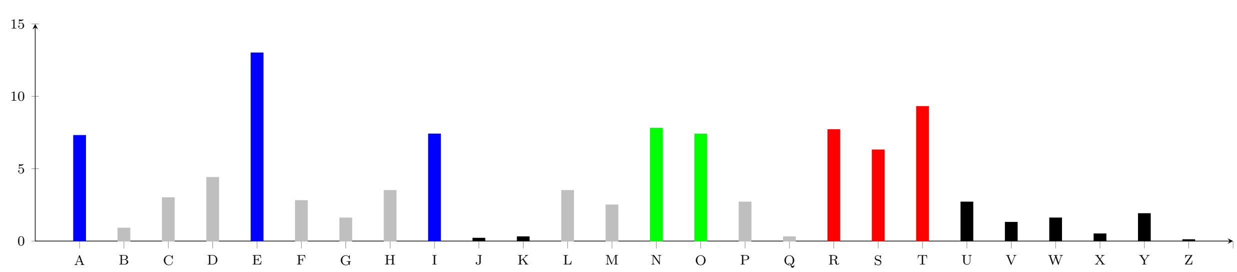
and the index of coincidence is just the sum over all possible letters:

$$\mathbf{IC}(\mathrm{Y}) = \sum_i \frac{f_i\cdot(f_i-1)}{N\cdot(N-1)} \ .$$

I.C. approximates the probability that any two letters randomly sampled from a distribution will be the same. Since IC approximates $\sum_i p_i^2$, it has the same range of variation 0.038 to 0.066, which corresponds to the sum of squares of the characteristic frequencies of English characters. The lower bound corresponds to a uniform distribution, and the upper bound correpends to monoalphabeticity.

On average, in a 1000 letter long sample of English text, the letters are distributed as follows:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | 73 | $B$ | 9 | $C$ | 30 | $D$ | 44 | $E$ | 130 | $F$ | 28 |
| $G$ | 16 | $H$ | 35 | $I$ | 74 | $J$ | 2 | $K$ | 3 | $L$ | 35 |
| $M$ | 25 | $N$ | 78 | $O$ | 74 | $P$ | 27 | $Q$ | 3 | $R$ | 77 |
| $S$ | 63 | $T$ | 93 | $U$ | 27 | $V$ | 13 | $W$ | 16 | $X$ | 5 |
| $Y$ | 19 | $Z$ | 1 | | | | | | | | |



The same picture would result from the examination of any reasonably long plain language text. Relative frequencies may vary slightly, but the basic facts remain the same:

- Evenly spaced vowels `A E I` with high frequency are evenly spaced 4 letters apart.
- Letter `E` is the most frequent of all the letters
- Consecutive part `N,O` have high frequency
- Consecutive triplet `R,S,T` has high frequency
- The pair `J,K` has low frequency
- The string `U,V,W,X,Y,Z` has low frequency.

## 2 Tasks

1. An additive cipher maps plaintext $G$ to ciphertext $X$. What is the encryption key? Which decryption key will allow to reconstruct the plaintext?
2. We know that a ciphertext was produced by a shift cipher, and that the encryption key was 17. What is the decryption key?
3. We know that the plaintext word `THE` is encrypted by an affine cipher into trigam `NHM`. What is the encryption key? What is the decryption key?
4. A ciphertext obtained by an affine cipher with key $(3, 17)$. Which key will you use to decrypt it?
5. What is the I.C. of the ciphertext `EPYEPOPDZSZUFPO`?
6. Encrypt the word `MORNING` using a shift cipher with key 11.
7. Encrypt the word `SYMBOL` using an affine cipher with key $(3, 2)$.
8. Encrypt the word `PARADOX` using a Vigenère cipher with key `YESTERDAY`.