

# Knowledge representation

## lecture 7

### Some applications of first order provers

<https://i.redd.it/4i4uv8qsh7yz.gif>

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# Lecture overview

Recall: three main kinds of reasoners (there are others ...)

An overview of using first order provers: where and how

Using provers in various kinds of mathematics

Nontrivial examples with Otter

Formal verification

Using provers for commonsense reasoning in natural language processing

# Three main kinds

- First order aka predicate logic (for classical logic)
- Propositional logic
- SMT (satisfiability modulu theories): prop logic + limited, computable parts of first order logic

# About propositional and SMT

Juhan Ernits will talk about the applications in the next lecture

# First order logic applications

Very rarely used in actual commercial software development.

Why: part of AI, not something you can easily use for creating a user interface or reports etc.

# First order logic and learning?

What makes logic hard: writing rules.

Normal people have really hard time encoding knowledge in formal rules.

Additionally, real-life rules are probabilistic/fuzzy, not certain.

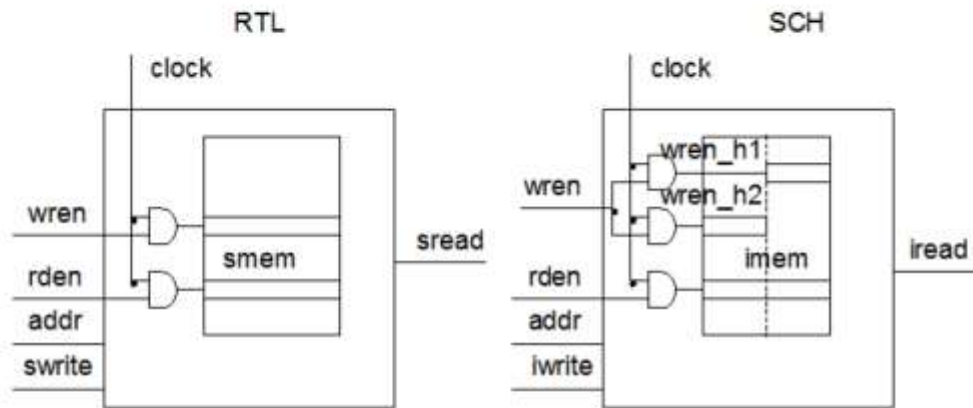
Hence, real hope or need is to start learning rules!

Once we have learned lots of rules, we could start using the rules, i.e. deriving new information.

# Automated Reasoning

- What is Reasoning? Solving problems by syntactic manipulations.

Hardware: Are these two hardware designs equivalent?



Software: Does your program accesses unallocated memory?

Math: Does this equation  $(xy)^{-1} = y^{-1}x^{-1}$  hold in all groups?

Knowledge management:

Can we represent and analyse all available knowledge about human body ?

Automated reasoning: can we solve all these problems automatically ?

# The scale of automation required

Intel floating arithmetic bug cost \$475 million.



Intel i7 Haswell-E 2,600,000,000 gates

Software bugs cost billions.



40 Mil LOC

**Major companies:** Intel, Microsoft, Airbus, NASA intensively use formal methods.



# In mathematics

Erdős discrepancy problem proved by a SAT solver (2014):



12GB proof

Largest math proof ever: Pythagorean triples problem was proved by a SAT solver (2016)



200TB proof

# Applications of automated reasoning

## Applications:

- software and hardware verification:  
Intel, Microsoft
- information management:  
biomedical ontologies,  
semantic Web, databases
- combinatorial reasoning: constraint  
satisfaction, planning, scheduling
- Internet security
- Theorem proving in mathematics



John McCarthy

“It is reasonable to hope that the relationship between computation and mathematical logic will be as fruitful in the next century as that between analysis and physics in the past.” McCarthy, 1963.

# Realities

Provers are – in an important sense – incredibly weaker than a brain.

Real mathematics (what mathematicians do) is very hard. Provers have managed to solve open problems only some rare cases.

Even mathematics is very hard to formalize, though doable.

Formalizing electronics is sometimes feasible, but really hard. In these cases prop solvers and SMT solvers work best.

We do not understand well how to formalize common sense thinking.

Common sense thinking relies on an enormous amount of uncertain rules.

# Hope!

Maybe we can learn commonsense rules using specialized learning techniques.

In parallel, maybe we can start automatically reading simple texts and automatically building statistical rules for commonsense reasoning.

After that we could use automated reasoning as a method for applying statistical rules we have learned.

# Examples from mathematics

Can we axiomatize „all mathematics“ at once?

Two answers:

- Gödel taught us that mathematics is not finitely axiomatizable.
- However, very large sensible parts have been axiomatized. See eg set theory in first order logic:
  - [https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel\\_set\\_theory](https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel_set_theory)
  - <http://www.cs.miami.edu/~tptp/cgi-bin/SeeTPTP?Category=Axioms&File=SET009^0.ax>
  - <http://www.cs.miami.edu/~tptp/cgi-bin/SystemOnTPTP> look for SET...

# TPTP

TPTP: the largest existing set of formalized first order problems

<http://www.cs.miami.edu/~tptp/>

along with

- converters
- an online tool capable of running all the top-level provers over the web interface.
- yearly prover competitions <http://www.cs.miami.edu/~tptp/CASC/>
- see also <http://www.satcompetition.org/> for SAT competitions

# Some nontrivial examples for otter

Let us examine

<http://lambda.ee/w/images/0/03/Problems.tar.gz>

Check these recommendations:

[http://lambda.ee/wiki/Soovitusi\\_Otteri\\_otsingu\\_suunamiseks](http://lambda.ee/wiki/Soovitusi_Otteri_otsingu_suunamiseks)

# Classic otter prover doing math

- Old results:

[http://www.mcs.anl.gov/research/projects/AR/new\\_results/](http://www.mcs.anl.gov/research/projects/AR/new_results/)

- Famous result: open problem - Robbins algebra ... – solved

<http://www.cs.unm.edu/~mccune/papers/robbins/>

- „See Otter digging for algebraic pearls“:

[http://www.math.md/files/qrs/v10-n1/v10-n1-\(pp95-114\).pdf](http://www.math.md/files/qrs/v10-n1/v10-n1-(pp95-114).pdf)



# Induction!?

Let us define the properties of a list member function  $m$  where  $c$  adds a new element to the list.  $n$  stands for empty list and a list  $[1,5]$  is represented as  $c(1, c(5, n))$

$\neg m(X, n)$ .

$m(X, c(X, Y))$ .

$X=Z \mid m(X, Y) \mid \neg m(X, c(Z, Y))$ .

$\neg m(X, Y) \mid m(X, c(Z, Y))$ .

Your goal is to prove that  $m$  has the expected property;

All  $x$  All  $y$  Exists  $z$  All  $u$   $((m(u, x) \ \& \ m(u, y)) \iff m(u, z))$

# Induction!?

We need to prove two statements:

Induction base:

All  $y$  Exists  $z$  All  $u$   $((m(u,n) \ \& \ m(u,y)) \Leftrightarrow m(u,z))$   
where  $n$  stands for empty list

Induction step:

All  $x$   $((\text{All } y \text{ Exists } z \text{ All } u ((m(u,x) \ \& \ m(u,y)) \Leftrightarrow m(u,z)))$   
 $\Rightarrow$   
 $(\text{All } h \text{ All } y1 \text{ Exists } z1 \text{ All } u1 ((m(u1,c(h,x)) \ \& \ m(u1,y1))$   
 $\Leftrightarrow m(u1,z1)))$

NB! There is no „easy“ way to guess what is a required base and step.

# Formal verification

# Verification vs. Simulation

$$(x+1)^2 = x^2 + 2x + 1$$

| X   | $(X+1)^2$ | $X^2+2X+1$ |
|-----|-----------|------------|
| 0   | 1         | 1          |
| 1   | 4         | 4          |
| 2   | 9         | 9          |
| ... | ...       | ...        |

|     |                            |
|-----|----------------------------|
| 1   | $(X+1)^2=(X+1)(X+1)$       |
| 2   | $(X+1)(X+1)=(X+1)X+(X+1)1$ |
| 3   | $(X+1)1=X+1$               |
| 4   | $(X+1)X=XX+1X$             |
| 5   | $XX=X^2$                   |
| ... | ....                       |

# Exhaustive Simulation Time

- Design: a 256-bit RAM.
  - $2^{256} = 10^{80}$  possible combinations of initial states and inputs
  - Assume:
    - Use all matter in our galaxy ( $10^{17}$  kg) to build computers.
    - Each computer is of the size of single electron ( $10^{-30}$  kg).
    - Each computer simulates  $10^{12}$  cases per second.
    - we started at the time of the Big Bang about  $10^{10}$  years ago
- ➔ We would just have reached the 0.05% mark of completing our task

# Detecting Design Faults

To check the new implementation for functional correctness, we need:

1. a reference description:
  - either a specification
  - or a previous “golden” implementation.
2. a new implementation, resulting from
  - a refinement (synthesis) of the specification
  - or an optimization of the reference implementation.
3. a correctness relation which has to be established between the two specifications
  - (e.g. behavioral correctness).

# Hardware Verification

Definition: Hardware verification

is the proof that a circuit or a system (the implementation) behaves according to a given set of requirements (the specification).

Formal verification

uses *mathematical reasoning* to prove that an implementation satisfies a specification

Consideration of *all cases is implicit* in formal verification.

# Formal Verification Methods

## Equivalence Checking

Compares optimized/synthesized model against original model

## Model Checking

Checks if a model satisfies a given property

## Theorem Proving

Proves implementation is equivalent to specification in some formalism



# What is Model Checking? (cont.)

Typically used during RTL code development to debug the RTL model prior to synthesis.

Used concurrently with and/or prior to simulation.

FormalCheck is the name of Cadence's Model Checking tool.

Currently the dominant formal verification tool.

# What is Model Checking?

Relies on exhaustive state space search.

exhaustive state space search is guaranteed to terminate, as the model is finite.

Major challenge:

To fight state-explosion problem

Can uncover subtle design errors

Can handle large state spaces ( $10^{120}$ )

Quicker to start testing

as it does not require vectors or a testbench.

Successfully used to find bugs in published standards

# What is Model Checking?

## Model Checking (Property Checking):

An automatic technique for verifying finite-state reactive systems, (such as sequential digital circuits or communication protocols).

A reactive FSM is an FSM whose inputs come from the environment.

For checking that a desired property holds in a finite state model of a system

Was pioneered by Edmund Clarke, professor in the CS Dept of CMU, in 1981

(E.M. Clarke and E.A. Emerson. "Synthesis of Synchronization Skeletons for Branching Time Temporal Logic", in Logic of Programs workshop, Yorktown Heights, NY, May 1981.).

# Applicability

## Wrong assumptions:

Formal methods can guarantee the perfectness of systems

## Facts:

can significantly enhance the trust, but not able to guarantee flawlessness.

## Reasons:

Only makes correctness statements with regard to a formal specification which can be faulty itself.

Faults in verification program.

Only design faults but not fabrication faults or faults during system usage.

# Some tools

## Verification Languages:

"e".

Synopsys Vera :

SystemC SCV

## Equivalence Checkers:

Cadence Verplex :

Synopsys Formality :

Mentor FormalPro :

Prover eCheck :

homebrew EC

## Assertion Languages:

IBM Sugar/PSL

0-in Checkerware

Verplex OVL

System Verilog SVA

Synopsys Vera OVA

# NLP processing

# NLP processing

„Simple“ parts can be done using either:

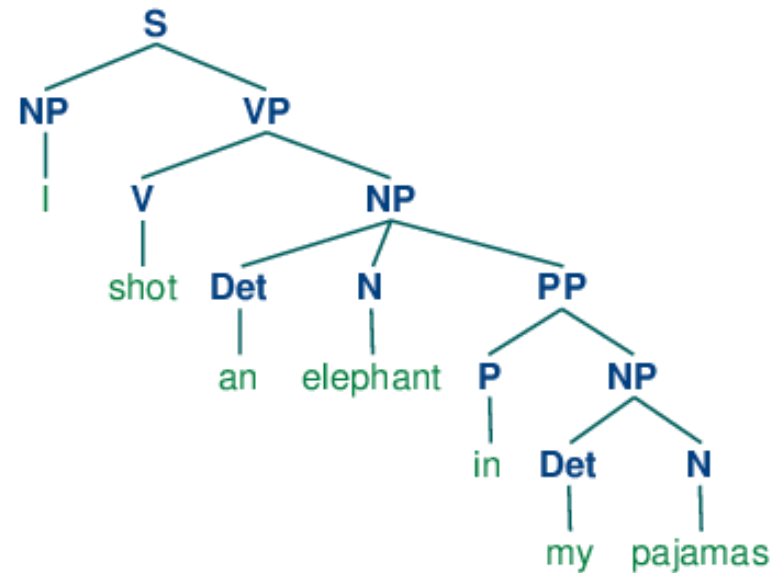
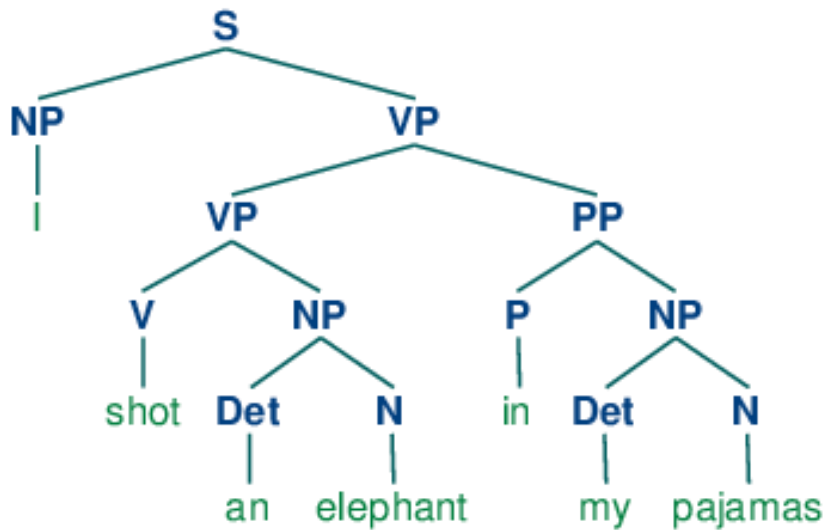
- „Simple“ rules (grammar, presence in Wikipedia, etc)
- Statistics and learned weight networks from large annotated text corpora

Hard parts require **commonsense understanding of the world:**

- Huge amounts of facts commonly known
- Huge amounts of probabilistic/fuzzy rules
- Weights for popularities, common usage patterns, etc

# Ambiguity in grammar

Different possible parses of the sentence „I shot an elephant in my pajamas.“





# Example

- Some roles prefer to be filled by certain kinds of NPs.
- This can give us useful features for classifying accurately:
  - “**John** ate the spaghetti with chopsticks.” (instrument)
  - “**John** ate the spaghetti with meatballs.” (patient)
  - “**John** ate the spaghetti with Mary.”
    - Instruments should be tools
    - Patient of “eat” should be edible
  - “**John** bought the car for \$21K.” (instrument)
  - “**John** bought the car for Mary.” (beneficiary)
    - Instrument of “buy” should be Money
    - Beneficiaries should be animate (things with desires)
  - “**John** drove Mary to school in the van”
  - “**John** drove the van to work with Mary.

# NLP processing: textual entailment

Means: commonsense derivations from text.

# Classical Entailment Definition

- Chierchia & McConnell-Ginet (2001):  
*A text  $t$  entails a hypothesis  $h$  if  $h$  is true in every circumstance (possible world) in which  $t$  is true*
- Strict entailment – doesn't account for some uncertainty allowed in applications

## “Almost certain” Entailments

- t: The technological triumph known as GPS ... was  
incubated in the mind of Ivan Getting.
- h: Ivan Getting invented the GPS.

# Applied Textual Entailment

- A directional relation between two text fragments:  
*Text (t)* and *Hypothesis (h)*:

*t entails h ( $t \Rightarrow h$ ) if  
humans reading *t* will infer that *h* is most likely true*

- Operational (applied) definition:
  - Human gold standard – as in NLP applications
  - Assuming common background knowledge – which is indeed expected from applications

# Probabilistic Interpretation

## Definition:

- *t* probabilistically entails *h* if:
  - $P(h \text{ is true} | t) > P(h \text{ is true})$ 
    - *t* increases the likelihood of *h* being true
    - $\equiv$  Positive PMI – *t* provides information on *h*'s truth
- $P(h \text{ is true} | t)$ : *entailment confidence*
  - The relevant entailment score for applications
  - In practice: “most likely” entailment expected

# The Role of Knowledge

- For textual entailment to hold we require:
  - $text \text{ AND } knowledge \Rightarrow h$
  - but
  - $knowledge$  should not entail  $h$  alone
- Systems are not supposed to validate  $h$ 's truth regardless of  $t$  (e.g. by searching  $h$  on the web)

# Gold standards

Annotated examples of text and sentence derivable from text:

Classical FRACAS example set:

<https://nlp.stanford.edu/~wcmac/downloads/fracas.xml>

Newer RTE example sets:

<https://tac.nist.gov/data/RTE/index.html>

Stanford Question Answering Dataset:

<https://rajpurkar.github.io/SQuAD-explorer/>



# **PASCAL Recognizing Textual Entailment (RTE) Challenges**

*EU FP-6 Funded PASCAL Network of Excellence  
2004-7*

***Bar-Ilan University***  
**MITRE**

**ITC-irst and CELCT, Trento**  
**Microsoft Research**

# Generic Dataset by Application Use

- 7 application settings in RTE-1, 4 in RTE-2/3
  - QA
  - IE
  - "Semantic" IR
  - Comparable documents / multi-doc summarization
  - MT evaluation
  - Reading comprehension
  - Paraphrase acquisition
- Most data created from actual applications output
- RTE-2/3: 800 examples in development and test sets
- 50-50% YES/NO split

# RTE Examples

|   | TEXT  | HYPOTHESIS  | TASK | ENTAILMENT |
|---|---|---|------|------------|
| 1 | <i>Regan attended a ceremony in Washington to commemorate the landings in Normandy.</i>   | <i>Washington is located in Normandy.</i>               | IE   | False      |
| 2 | <i>Google files for its long awaited IPO.</i>   | <i>Google goes public.</i>                              | IR   | True       |
| 3 | <i>....: a shootout at the Guadalajara airport in May, 1993, that killed Cardinal Juan Jesus Posadas Ocampo and six others.</i>         | <i>Cardinal Juan Jesus Posadas Ocampo died in 1993.</i> | QA   | True       |
| 4 | <i>The SPD got just 21.5% of the vote in the European Parliament elections, while the conservative opposition parties polled 44.5%.</i> | <i>The SPD is defeated by the opposition parties.</i>   | IE   | True       |

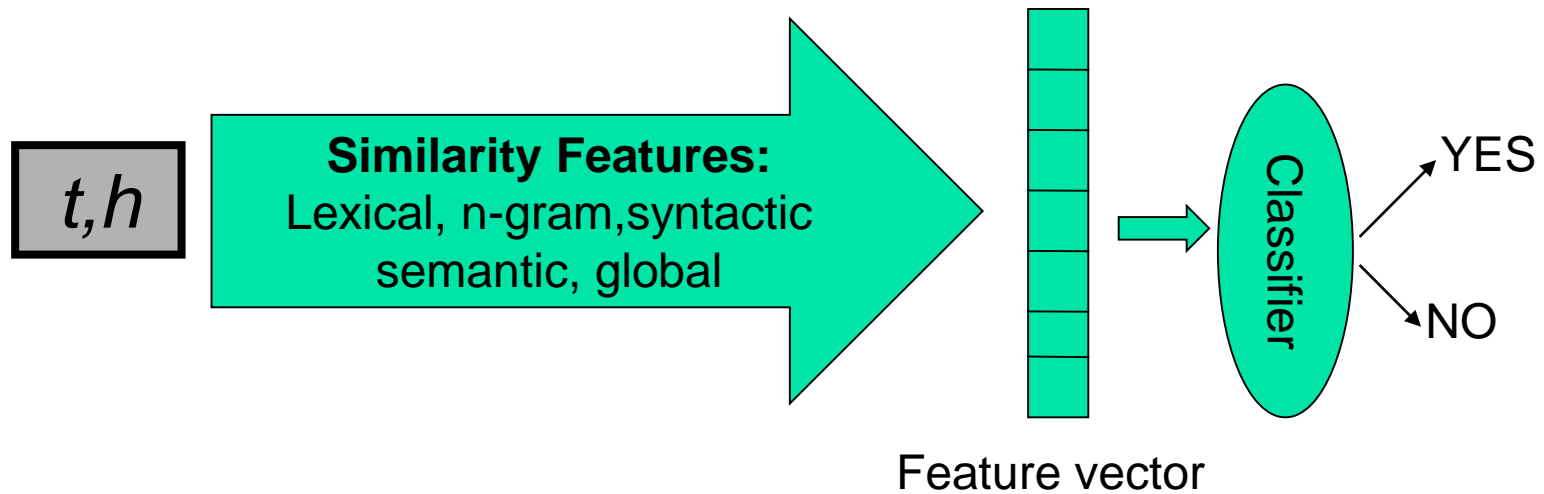
# Participation and Impact

- Very successful challenges, world wide:
  - RTE-1 – 17 groups
  - RTE-2 – 23 groups
    - ~150 downloads
  - RTE-3 – 25 groups
    - Joint workshop at ACL-07
- High interest in the research community
  - Papers, conference sessions and areas, PhD's, influence on funded projects
  - Textual Entailment special issue at JNLE
  - ACL-07 tutorial

# Methods and Approaches (RTE-2)

- Measure similarity match between  $t$  and  $h$  (*coverage of  $h$  by  $t$* ):
  - Lexical overlap (unigram, N-gram, subsequence)
  - Lexical substitution (WordNet, statistical)
  - Syntactic matching/transformations
  - Lexical-syntactic variations ("paraphrases")
  - Semantic role labeling and matching
  - Global similarity parameters (e.g. negation, modality)
- Cross-pair similarity
- Detect mismatch (for non-entailment)
- Interpretation to logic representation + logic inference

# Dominant approach: Supervised Learning



- Features model similarity and mismatch
- Classifier determines relative weights of information sources
- Train on development set and auxiliary  $t-h$  corpora

# RTE-2 Results

| First Author (Group)    | Accuracy    | Average Precision           |
|-------------------------|-------------|-----------------------------|
| Hickl (LCC)             | 75.4%       | 80.8%                       |
| Tatu (LCC)              | 73.8%       | 71.3%                       |
| Zanzotto (Milan & Rome) | 63.9%       | 64.4%                       |
| Adams (Dallas)          | 62.6%       | 62.8%                       |
| Bos (Rome & Leeds)      | 61.6%       | 66.9%                       |
| 11 groups               | 58.1%-60.5% | Average: 60%<br>Median: 59% |
| 7 groups                | 52.9%-55.6% |                             |