Lecture 5
Module I: Model Checking
Topic: Model checking timed transition systems: timed automata

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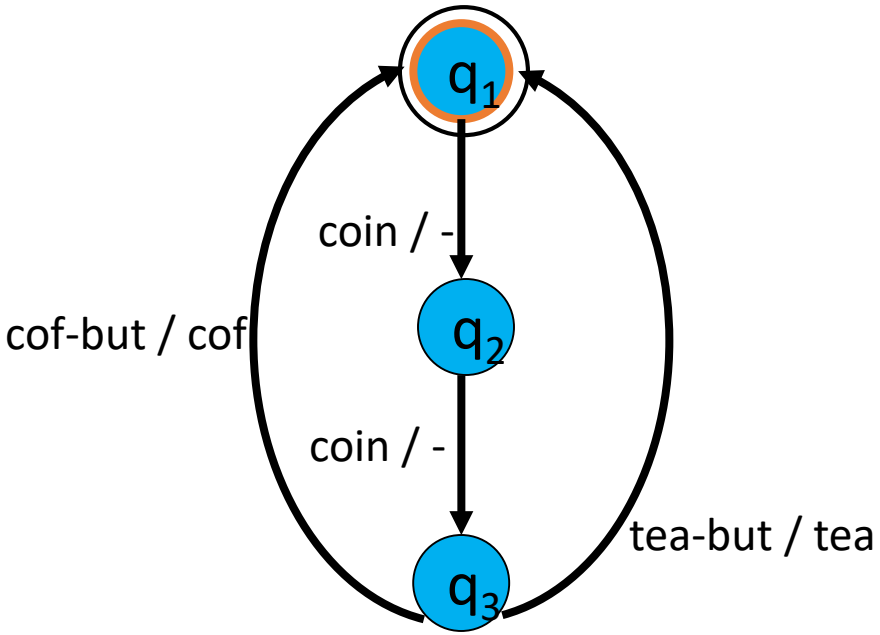
Slides by **Brian Nielsen** (AU)

Finite State Machine (Mealy)



condition		effect	
current state	input	output	next state
q_1	coin	-	q_2
q_2	coin	-	q_3
q_3	cof-but	cof	q_1
q_3	tea-but	tea	q_1

Inputs = {cof-but, tea-but, coin}
Outputs = {cof,tea}
States: {q_1,q_2,q_3}
Initial state = q_1
Transitions= {
(q_1, coin, -, q_2),
(q_2, coin, -, q_3),
(q_3, cof-but, cof, q_1),
(q_3, tea-but, tea, q_1)
}

Sample run:

q_1 —coin/ -→ q_2 —coin/-→ q_3 —cof-but / cof→ q_1 —coin/ -→

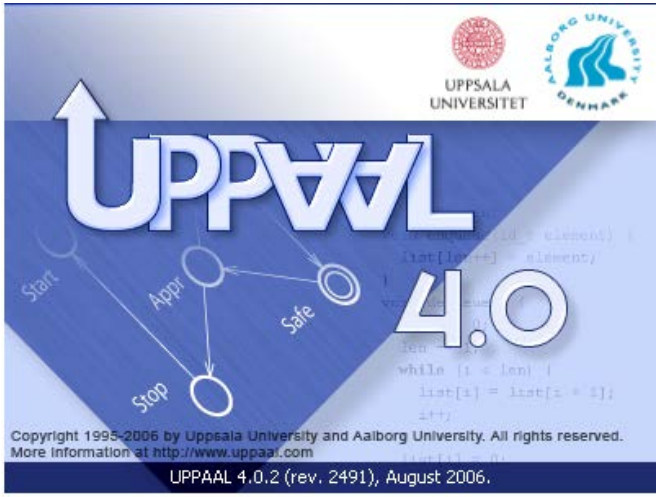
q_2 —coin/ -→ q_3 —cof-but / cof→ q_1

Adding Time

FSM

↓

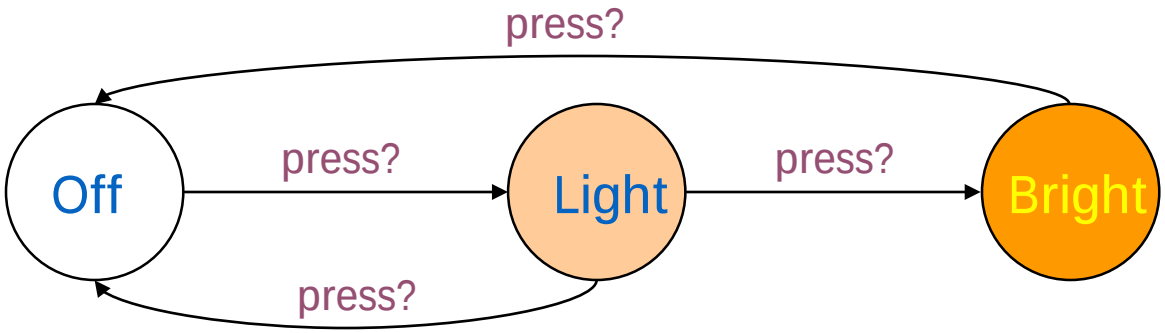
Timed Automata



Dumb Light Control

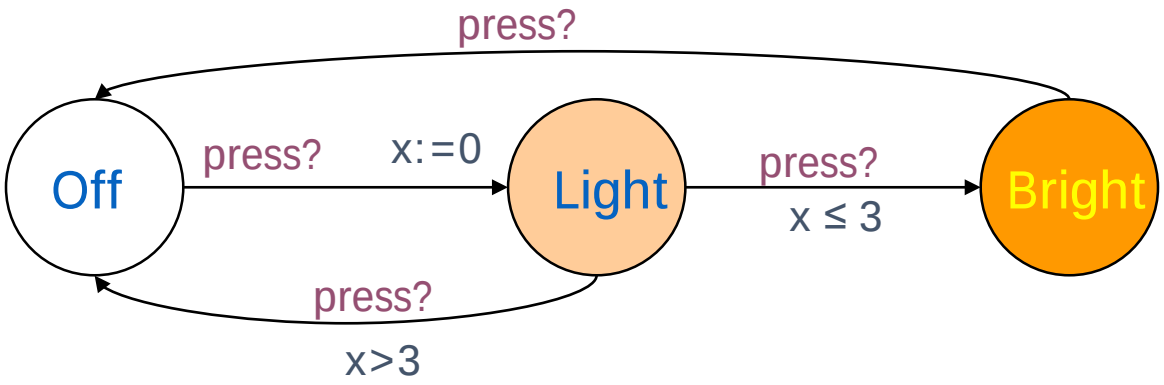
Reguirement: if press is issued twice quickly then the light will get brighter; otherwise the light is turned off.

Solution 1:

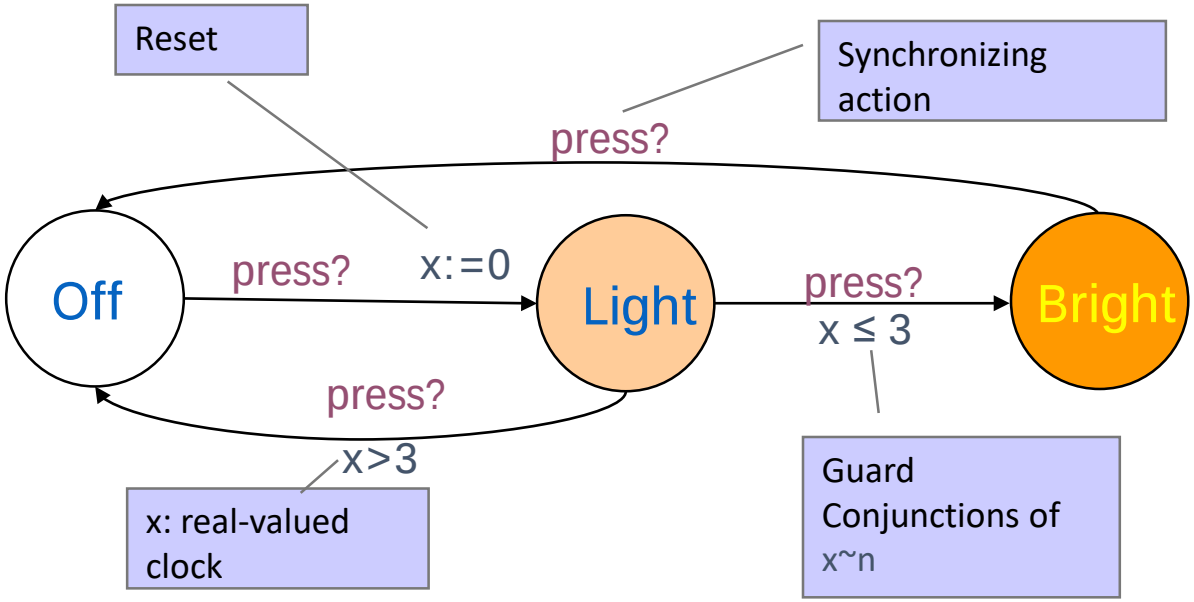


Dumb Light Control

Solution 2: Add real-valued clock x to model the timing requirements |[*quickly*]| = x ≤ 3



Timed Automata



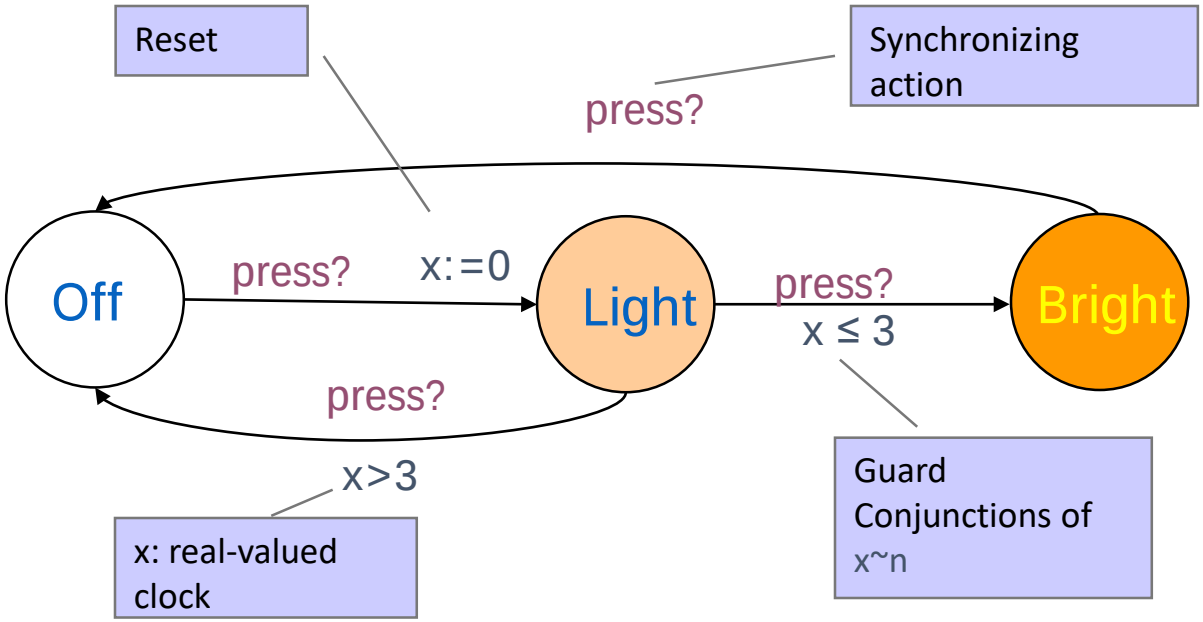
States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

Timed Automata



States:

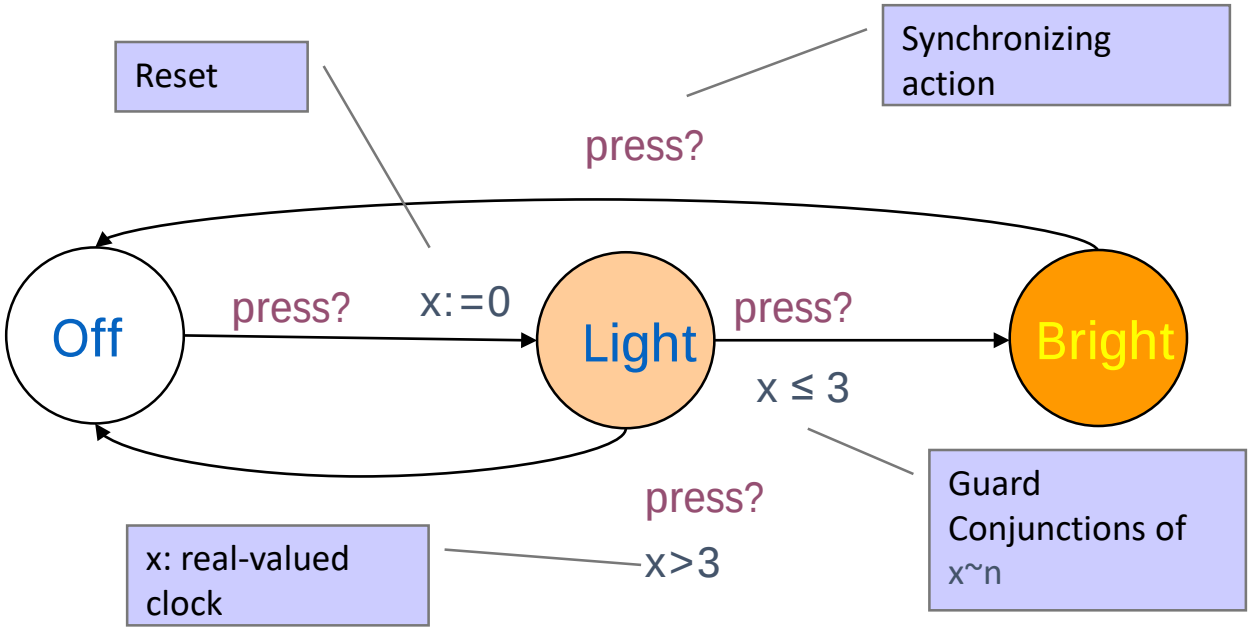
(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

Timed Automata



States:

(location , x=v) where v ∈ **R**

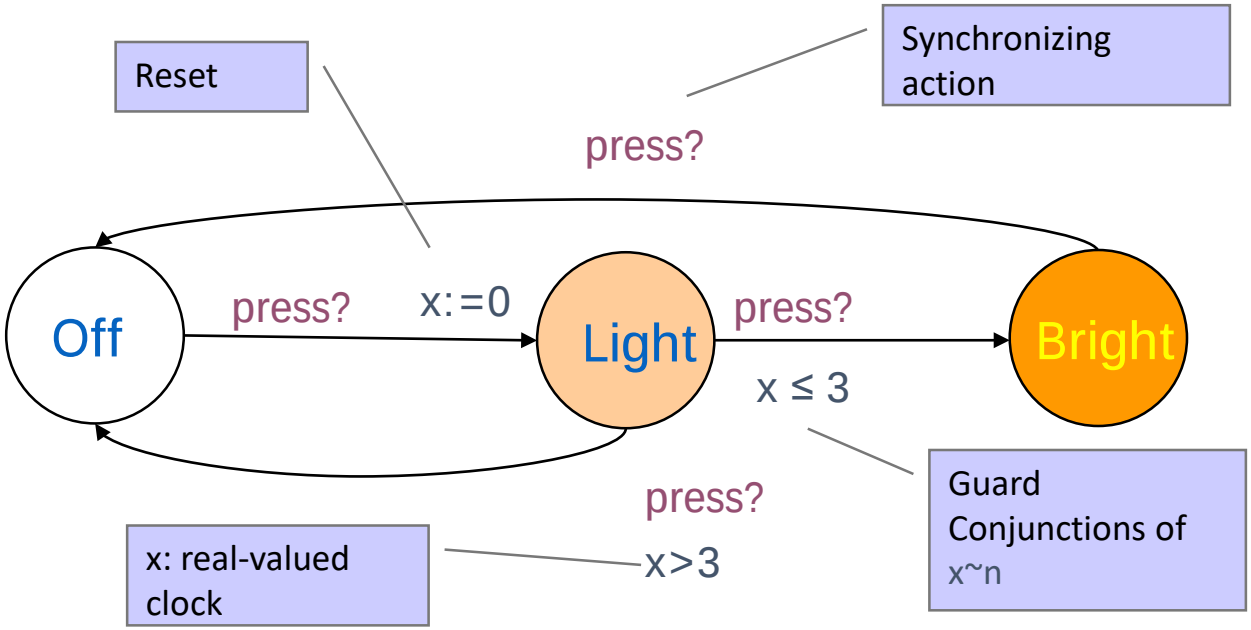
Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

press? → (Light , x=0)

Timed Automata



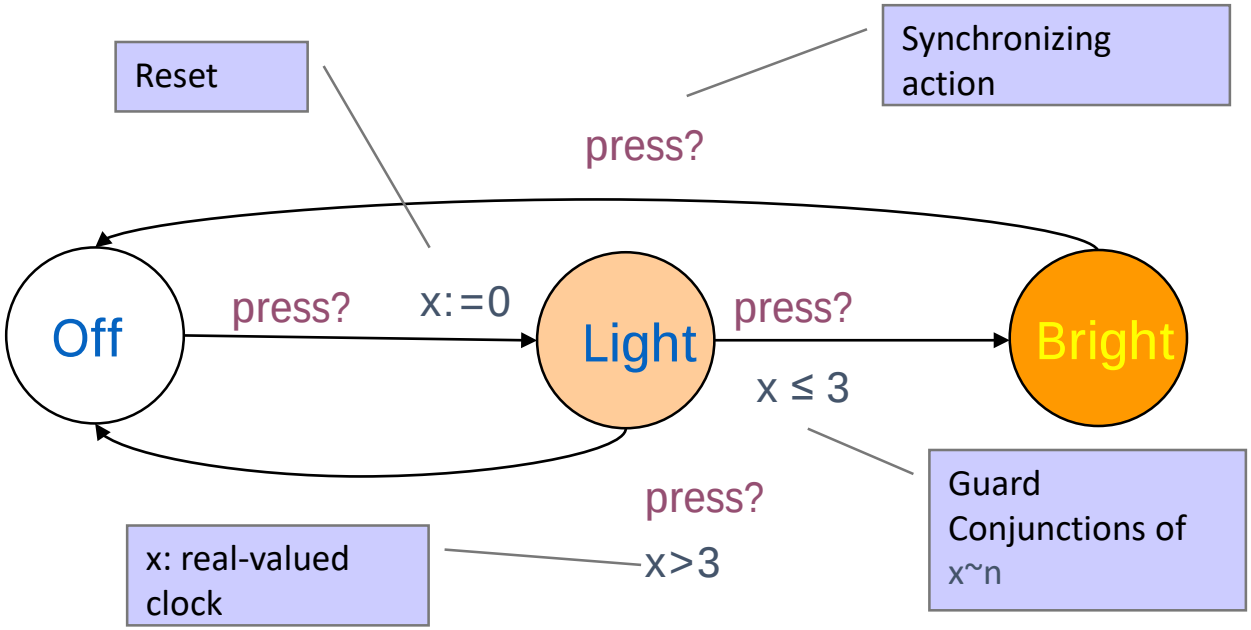
States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)
delay 4.32 → (Off , x=4.32)
press? → (Light , x=0)
delay 2.51 → (Light , x=2.51)

Timed Automata



States:

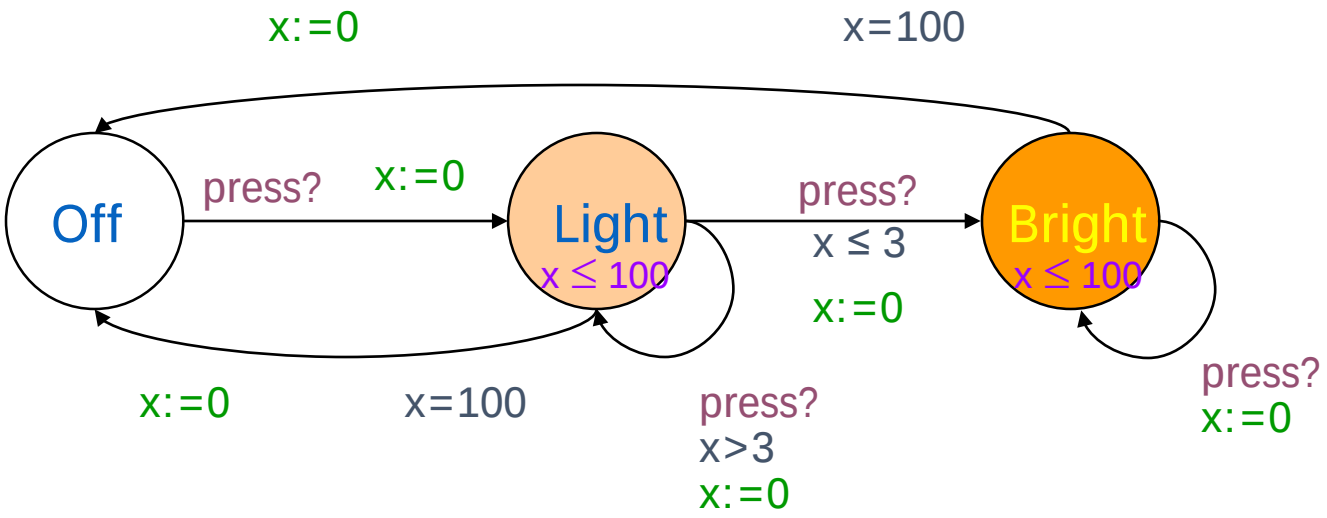
(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)
delay 4.32 → (Off , x=4.32)
press? → (Light , x=0)
delay 2.51 → (Light , x=2.51)
press? → (Bright , x=2.51)

Intelligent Light Control

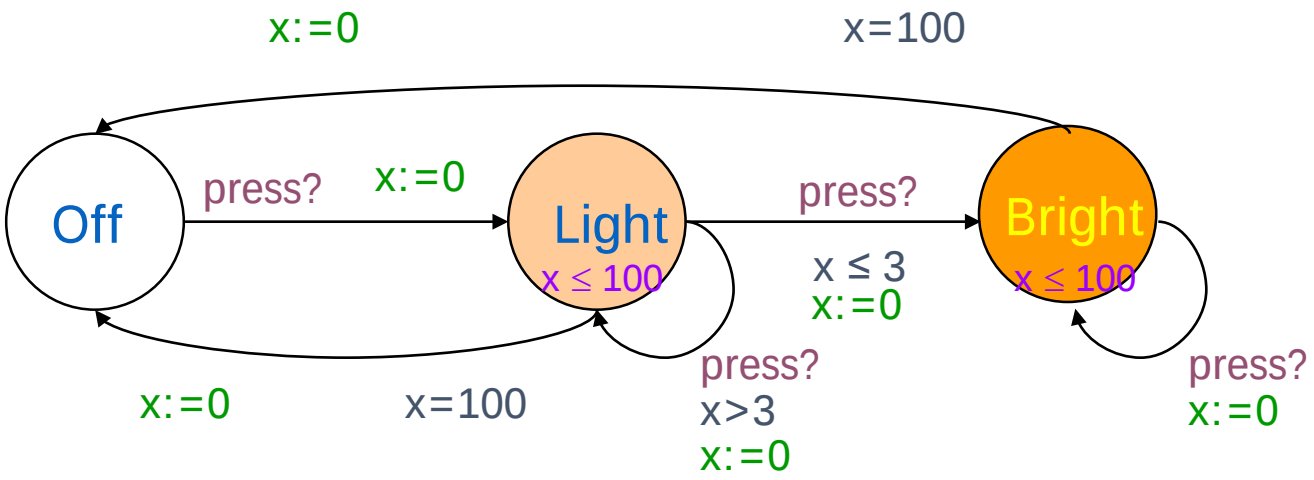
Reguirement: : automatically switch light off after 100 time units



Upper time bound is specified using invariants

Intelligent Light Control

Using Invariants



Transitions:

	(Off , x=0)
delay 4.32	→ (Off , x=4.32)
press?	→ (Light , x=0)
delay 4.51	→ (Light , x=4.51)
press?	→ (Light , x=0)
delay 100	→ (Light , x=100)
τ	→ (Off , x=0)

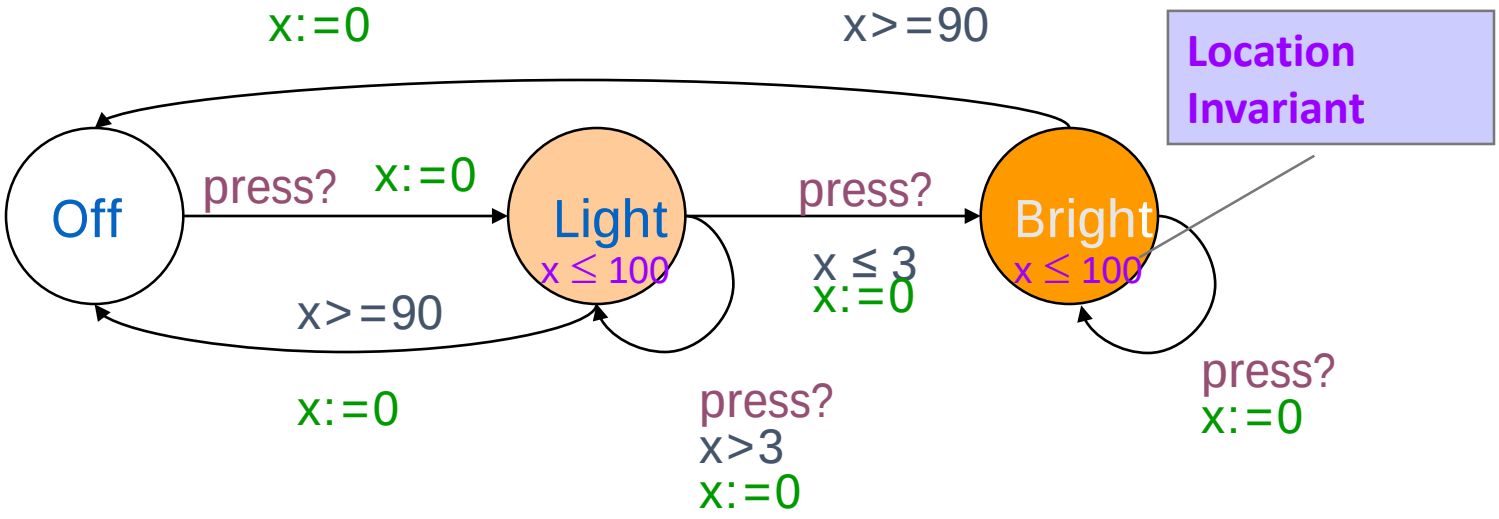
Note:
(Light , x=0) delay 103 →

X

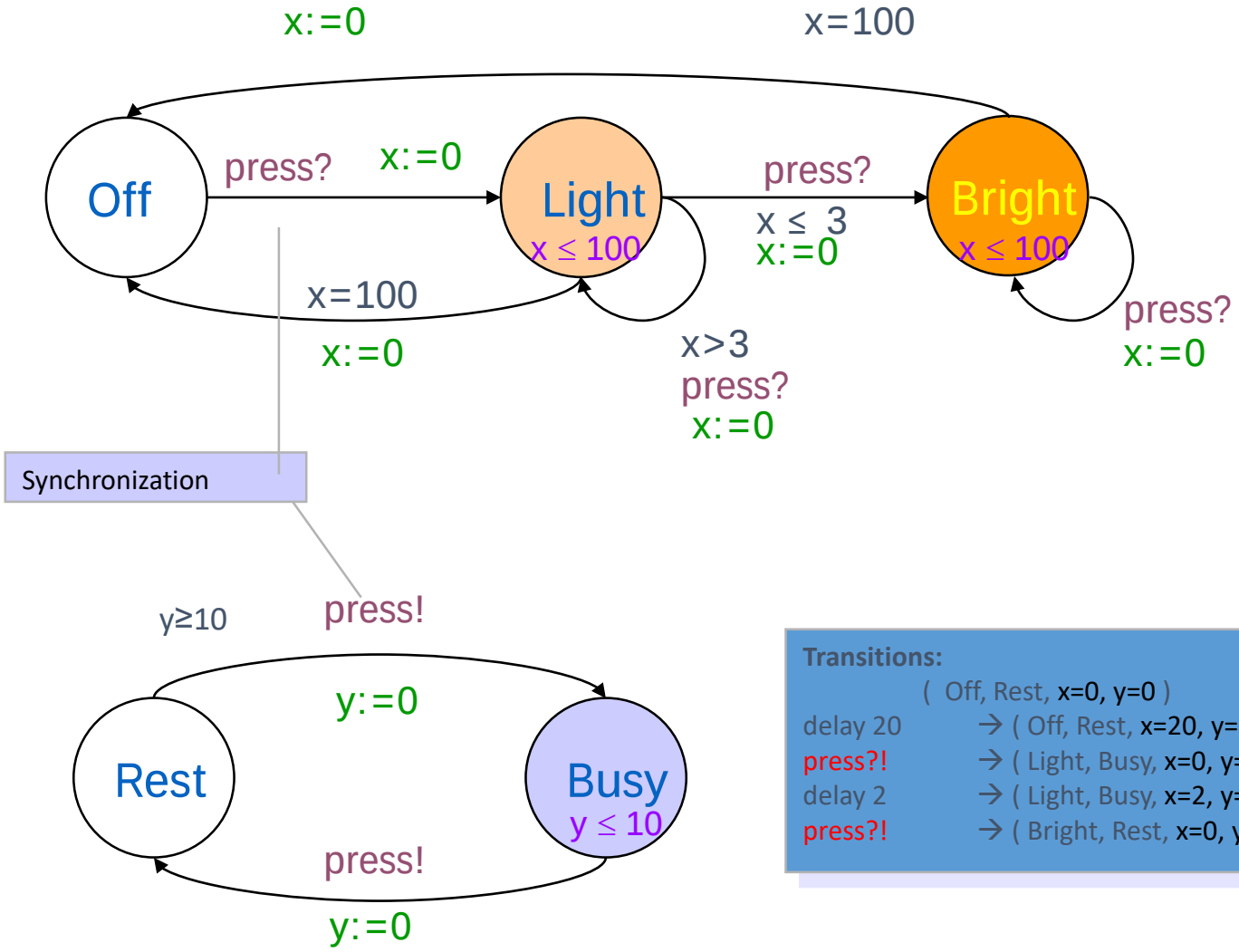
Invariants ensures progress

Intelligent Light Control

If requirements include **uncertainty**:
Automatically switch light off ***between*** 90-100 time units after switching on.



Light Controller || User

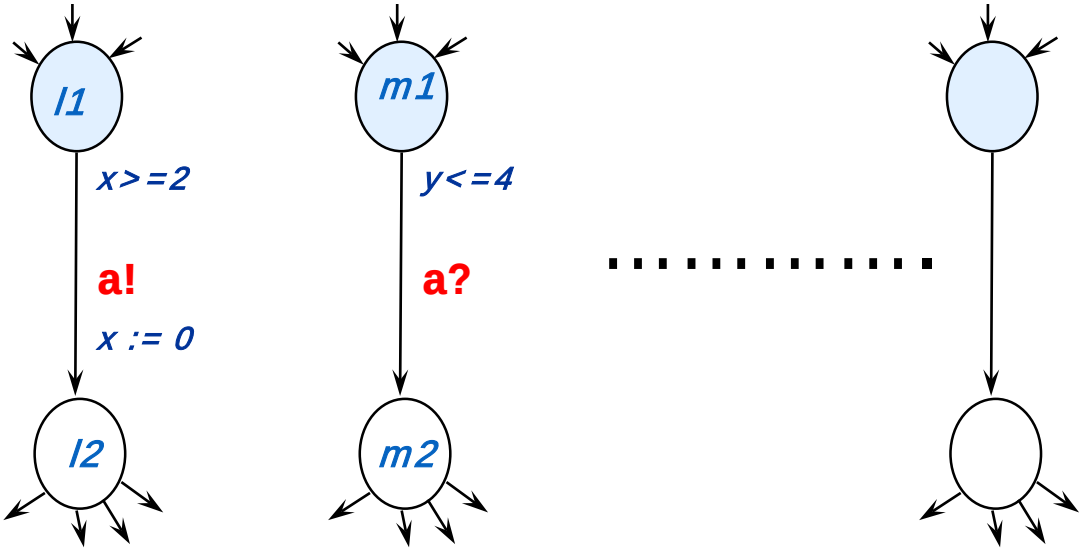


Transitions:

(Off, Rest, x=0, y=0)
delay 20 → (Off, Rest, x=20, y=20)
press?! → (Light, Busy, x=0, y=0)
delay 2 → (Light, Busy, x=2, y=2)
press?! → (Bright, Rest, x=0, y=0)

Networks of Timed Automata

(a'la CCS)



Two-way synchronization on *complementary* actions.

Closed Systems!

Example transitions

(*l1*, *m1*,....., x=2, y=3.5,.....) —tau→ (*l2*,*m2*,.....,x=0, y=3.5, ...)

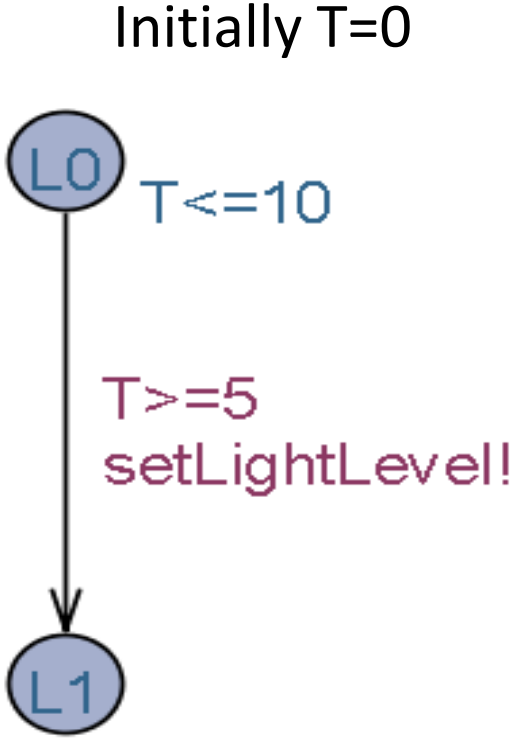
0.2 → (*l1*,*m1*,.....,x=2.2, y=3.7, ...)

If **a** URGENT CHANNEL

Timing Uncertainty

- Unpredictable or variable timing
  - response time,
  - computation time
  - transmission time etc:

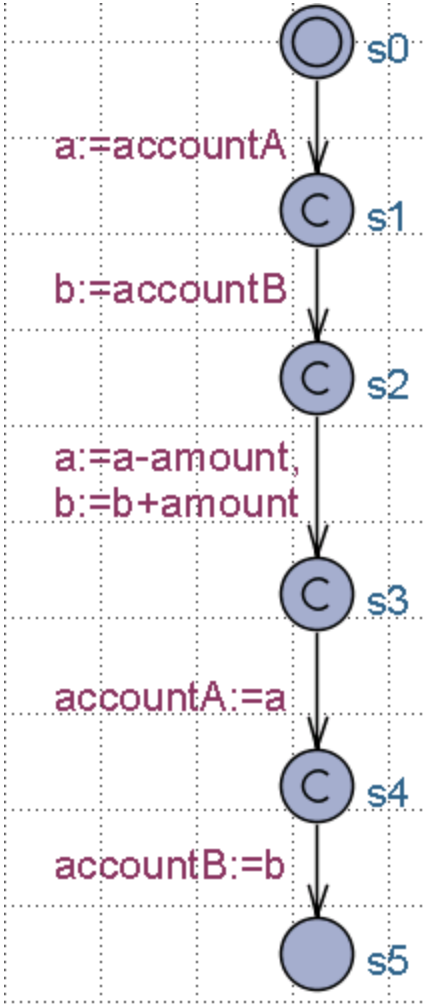
Example:
Light level must be adjusted
between 5 and 10 time units



Comitted Locations

- Locations marked **C**
  - ***No delay*** in committed location.
  - No interleaving with parallel transitions
- Handy to model atomic sequences
- The use of committed locations reduces the number of states in a model, and allows for more space and time efficient analysis.

- S0 to s5 executed atomically



Urgent Channels and Locations

- Locations marked **U**
  - ***No delay*** like in committed location.
  - But Interleaving permitted
- Channels declared "**`urgent chan`**"
  - Time doesn't elapse when a synchronization is possible on a pair of urgent channels
  - Interleaving allowed

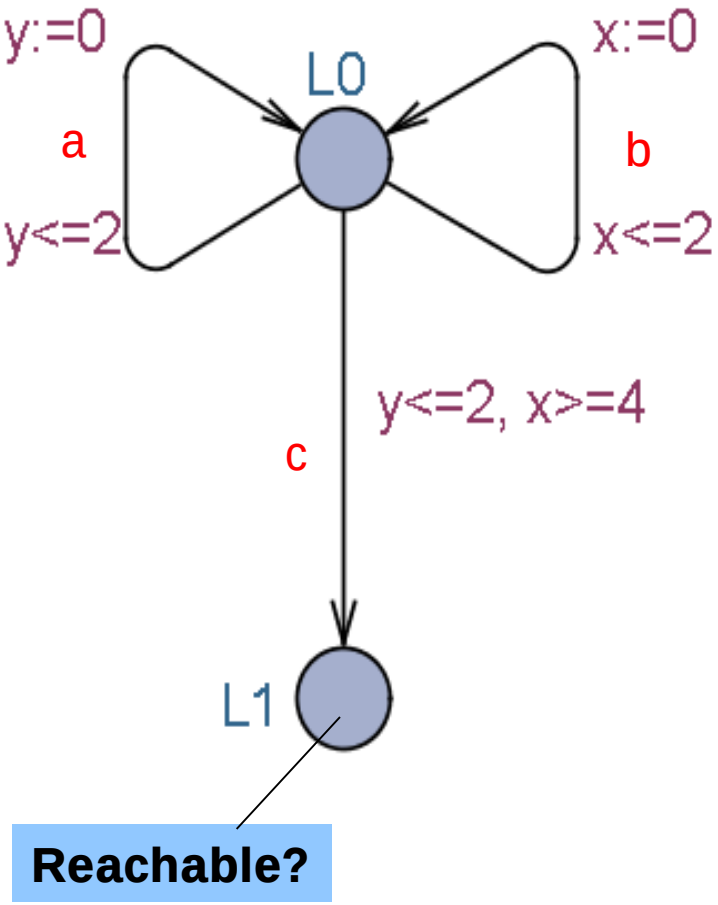
Broad-casts

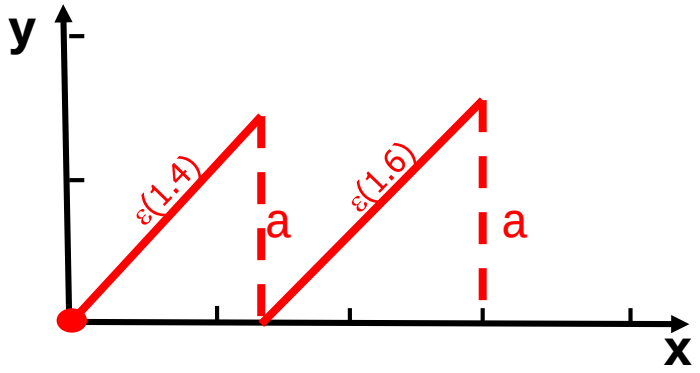
- broadcast channel:
  - sending: output chan!
  - every automaton that listens to join moves on
  - ie. every automaton with enabled “chan?” transition moves in one step
  - may be zero! Listeners, sender can progress anyway

Other Uppaal features

- Bounded domains
  - `int [1..4] a;`
- C-like data-structures and user defined functions in declaration section
  - structs, arrays, and typedef
- non-deterministic assignment:
  - `select a:T    // select a random value from T`
- `forall`, `exists` in expressions
- Scalar sets (for giving unique ID's)
- Process and channel **priorities**
- Value passing (emulation)

Semantics: Timed traces



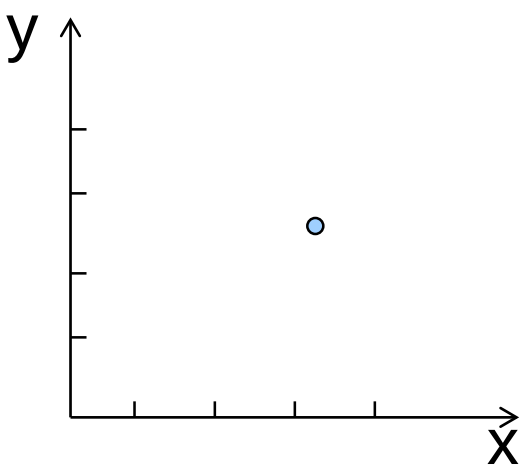


(L0,x=0,y=0)
→ε(1.4)
(L0,x=1.4,y=1.4)
→a
(L0,x=1.4,y=0)
→ε(1.6)
(L0,x=3.0,y=1.6)
→a
(L0,x=3.0,y=0)

From explicit clock values to zones (*from infinite to finite*)

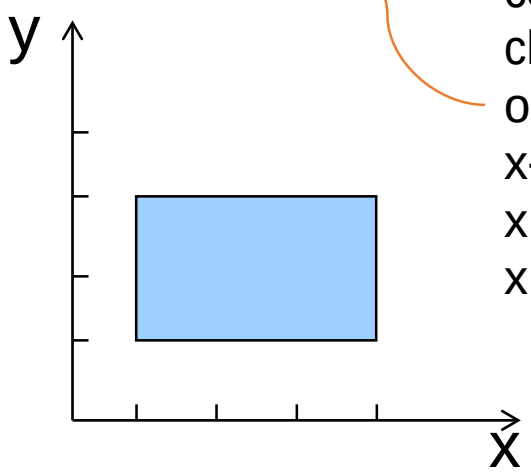
Explicit state

(n, x=3.2, y=2.5)



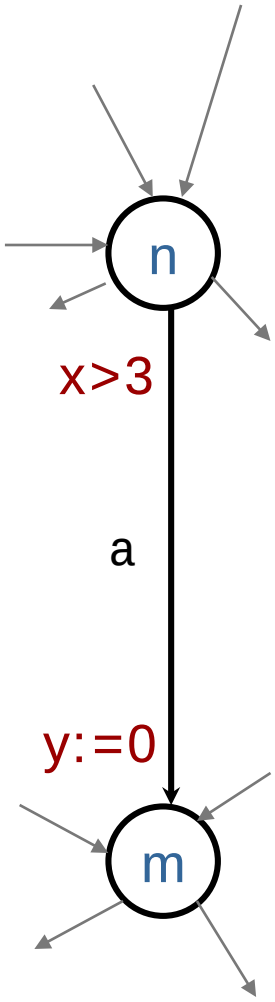
Symbolic state (set)

(n, $1 \leq x \leq 4, 1 \leq y \leq 3$)

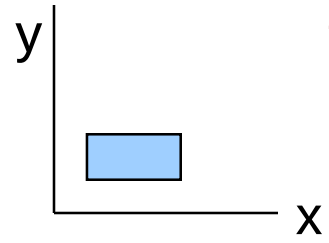


Zone:
conjunction of clock constraints of form:
x-y<=const1,
x<=const2,
x>=const3

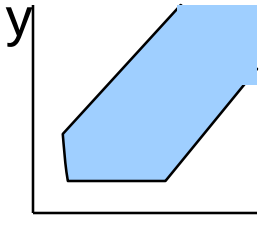
Symbolic Transitions



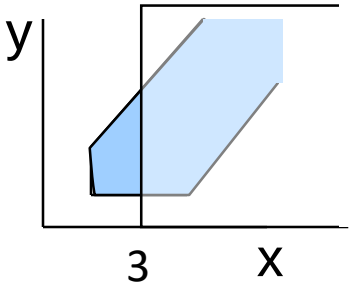
$1 \le x \le 4$
$1 \le y \le 3$



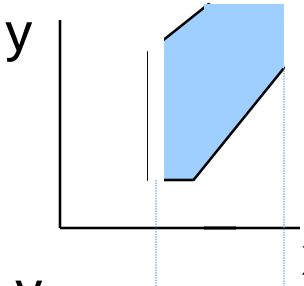
In n delays to



$1 \le x, 1 \le y$
$-2 \le x-y \le 3$

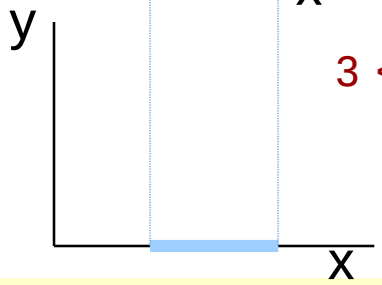


Guard conjoins to



$3 < x, 1 \le y$
$-2 \le x-y \le 3$

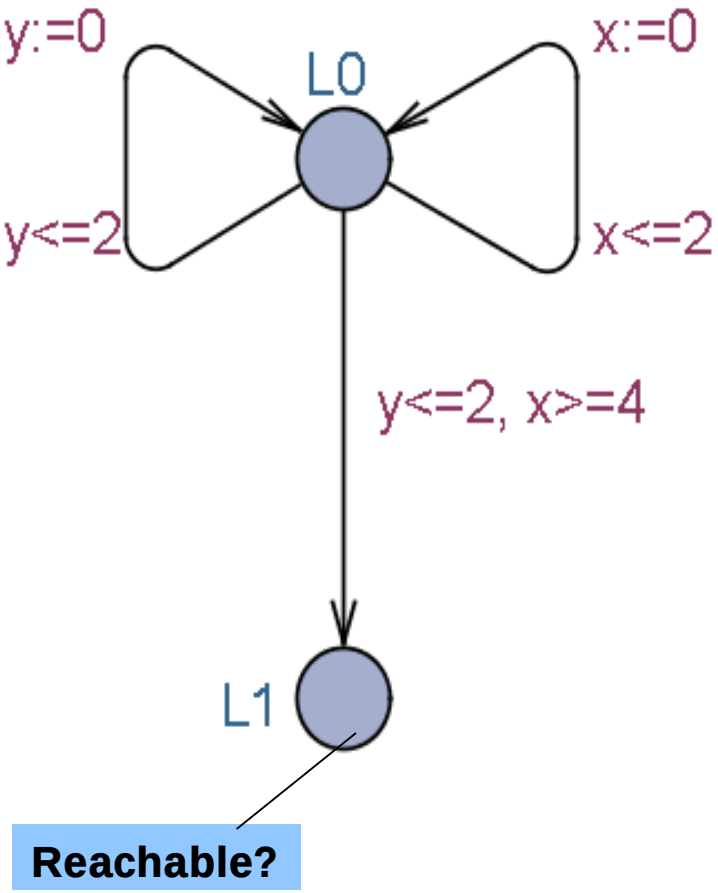
Update projects to

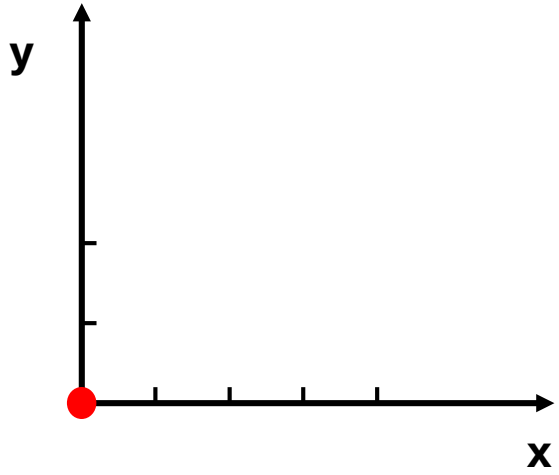


$3 < x, y=0$

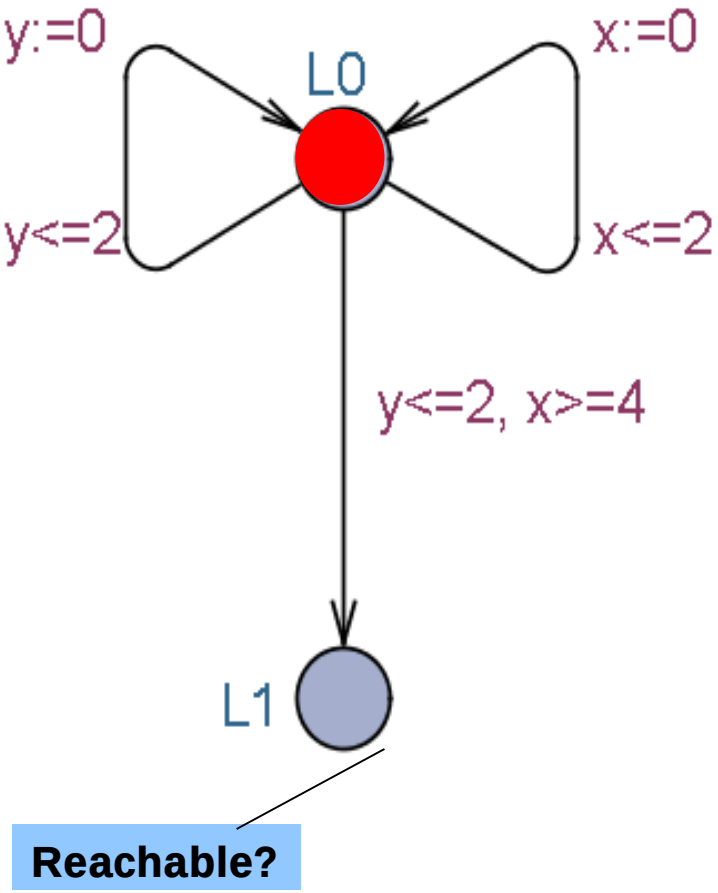
Thus $(n, 1 \le x \le 4, 1 \le y \le 3) \rightarrow^{a} (m, 3 < x, y=0)$

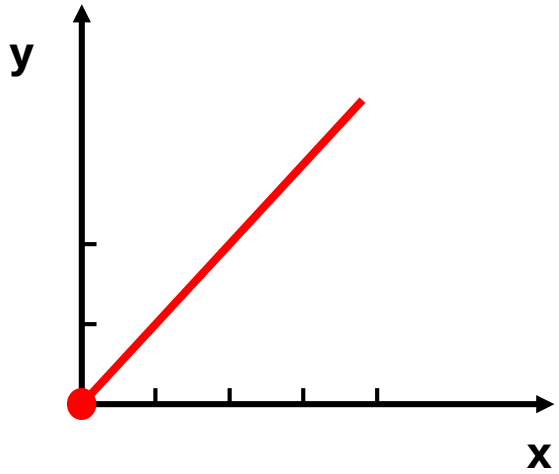
Symbolic state exploration





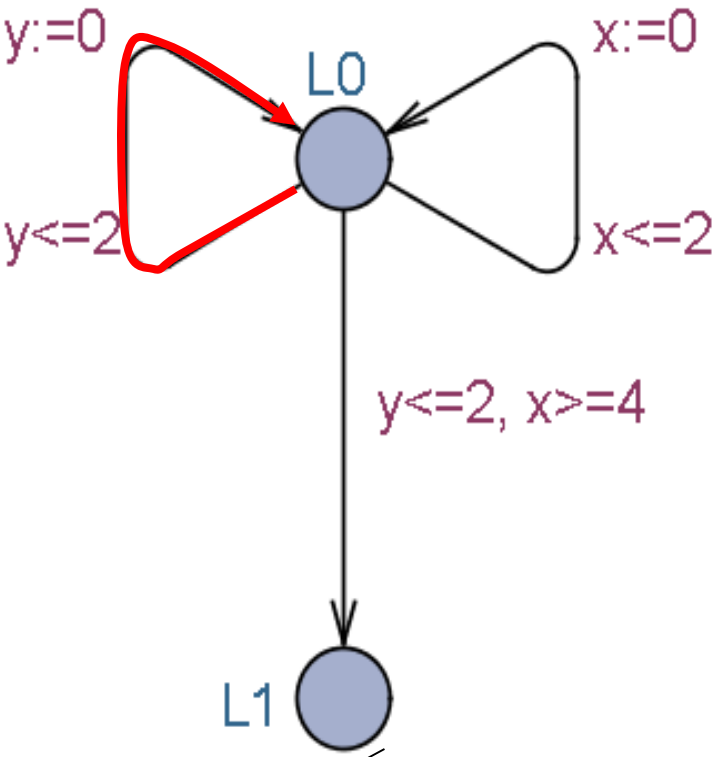
Symbolic Exploration

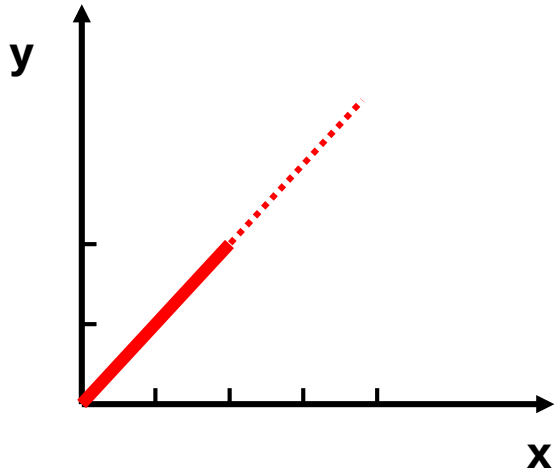




Delay

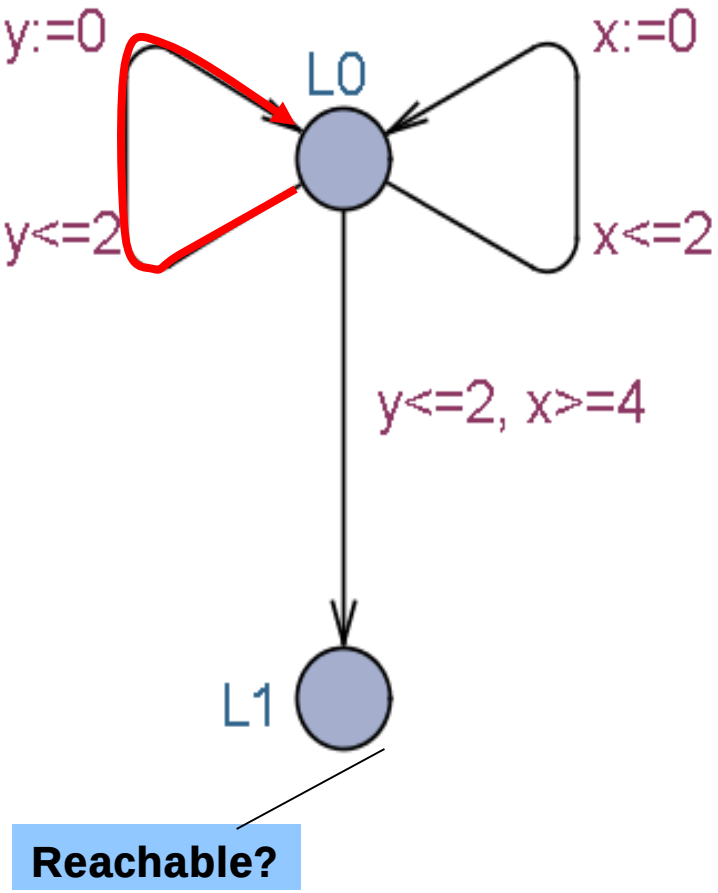
Symbolic Exploration

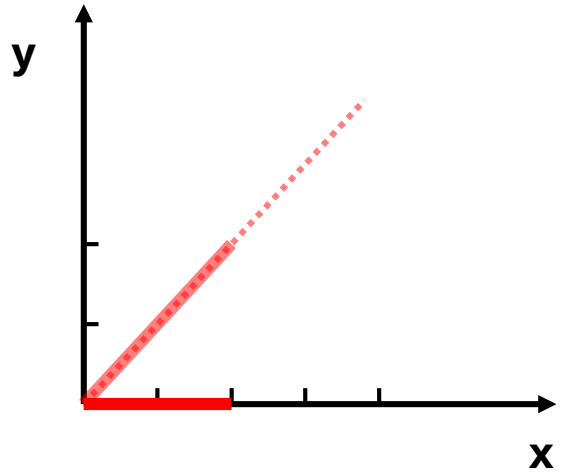




Left

Symbolic Exploration





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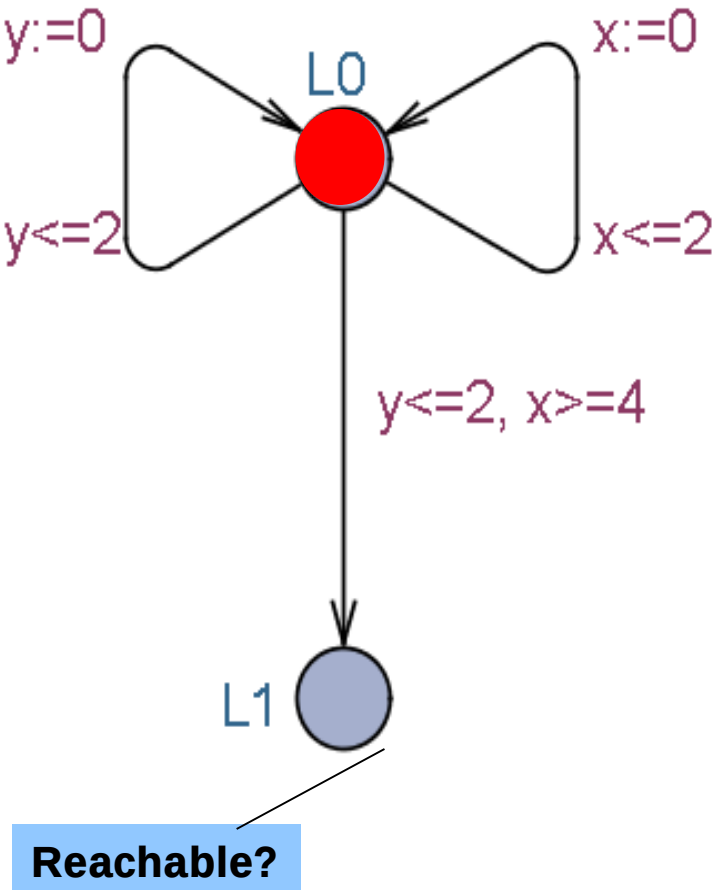
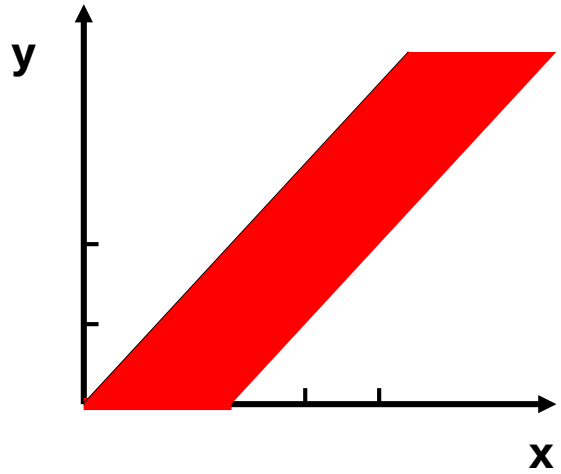
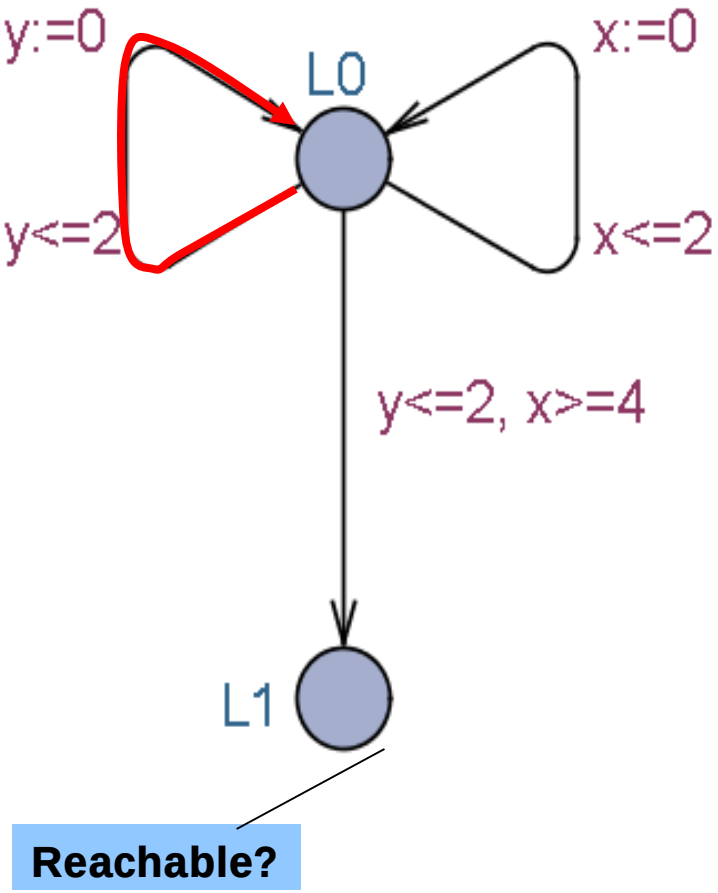
Symbolic Exploration

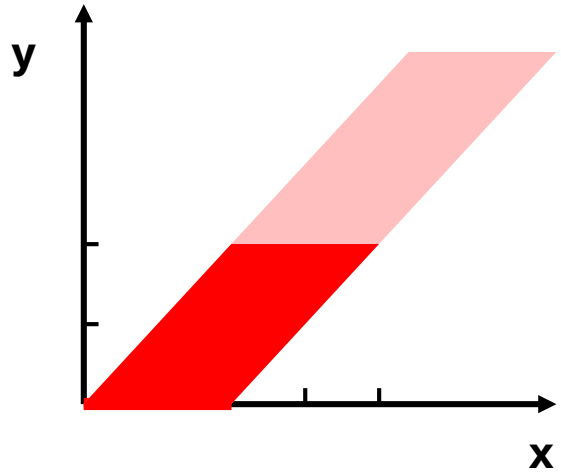




Delay

Symbolic Exploration





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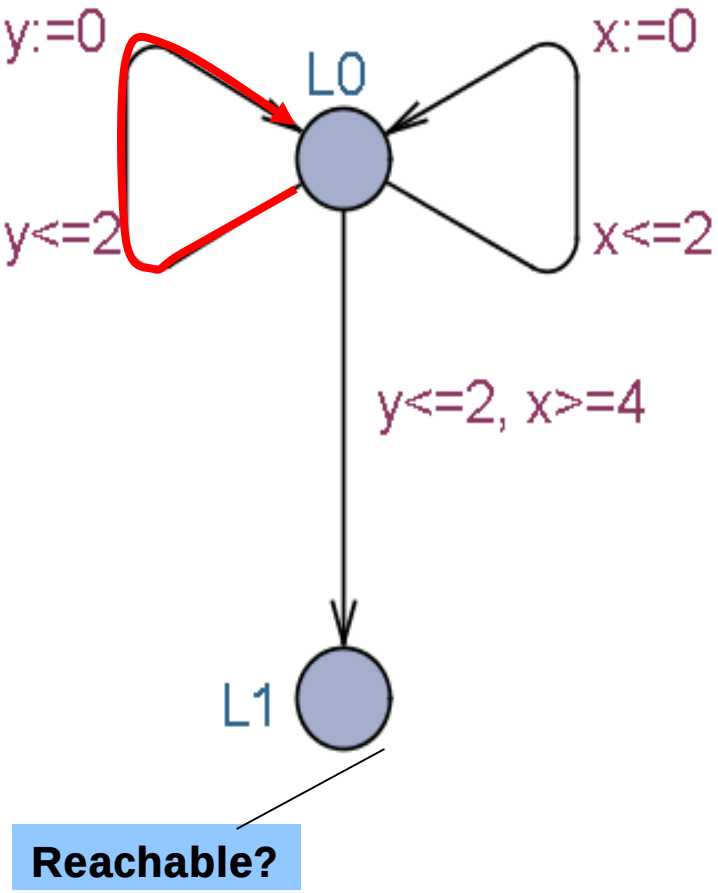
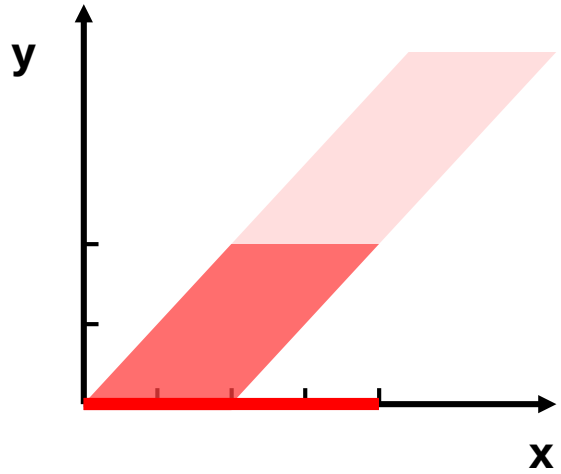
Symbolic Exploration





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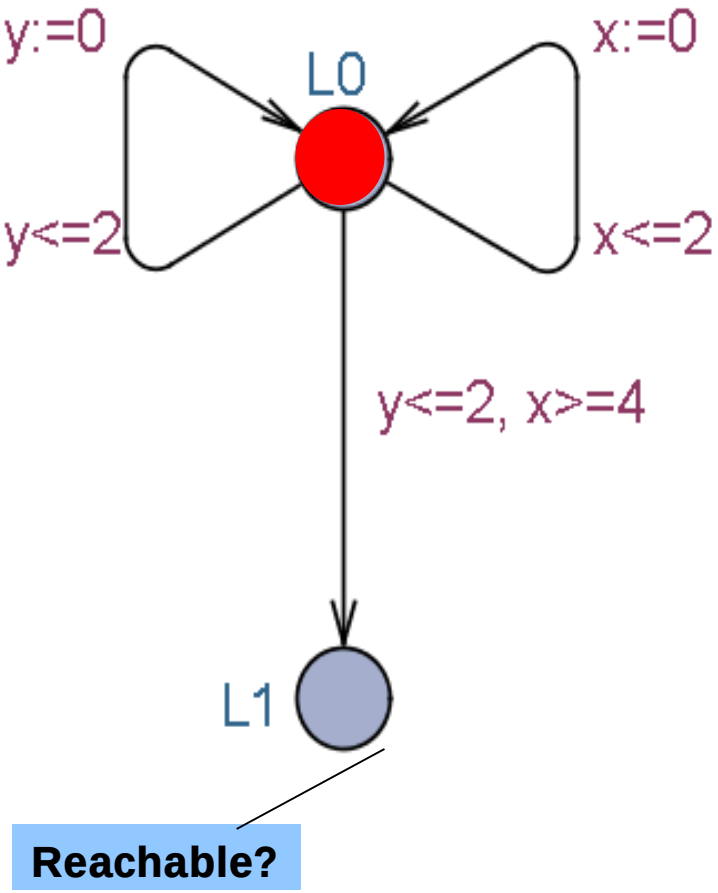
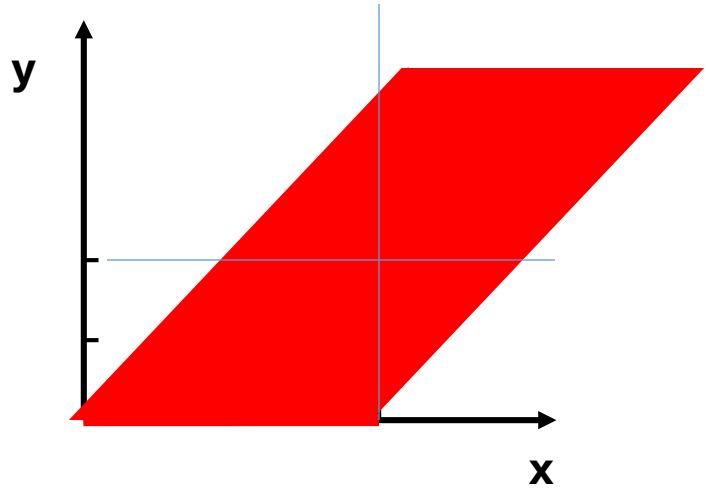
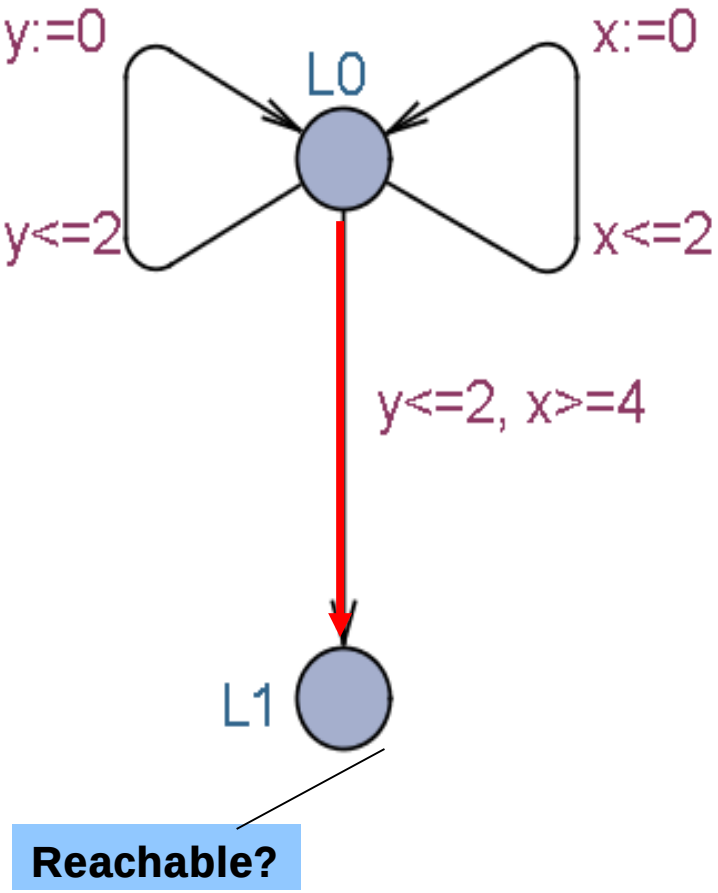
Symbolic Exploration

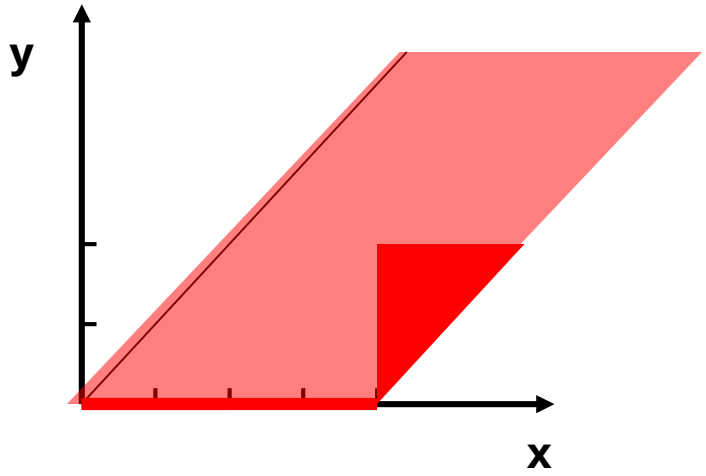




Delay

Symbolic Exploration



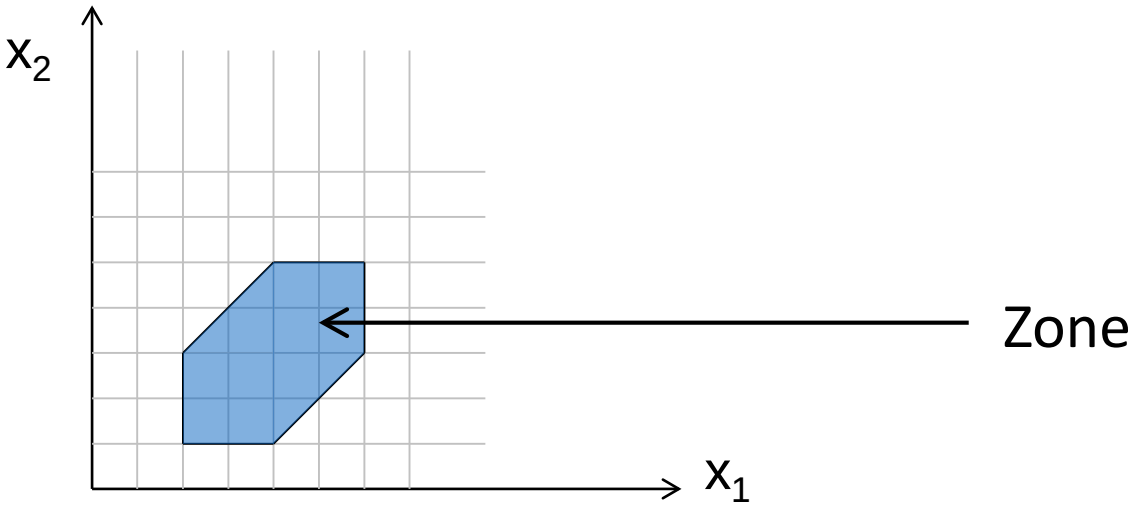


Down

Difference Bound Matrices

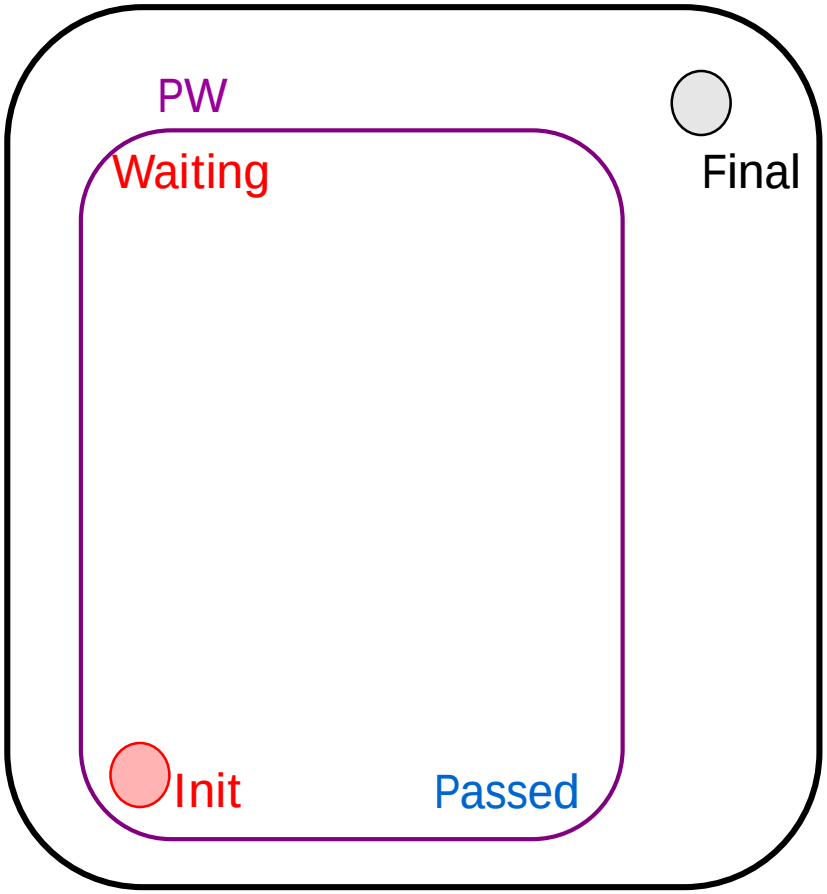
$x_0 - x_0 <= 0$	$x_0 - x_1 <= -2$	$x_0 - x_2 <= -1$
$x_1 - x_0 <= 6$	$x_1 - x_1 <= 0$	$x_1 - x_2 <= 3$
$x_2 - x_0 <= 5$	$x_2 - x_1 <= 1$	$x_2 - x_2 <= 0$

$$x_i - x_j <= c_{ij}$$



Forward Reachability Algorithm

Init -> Final ?



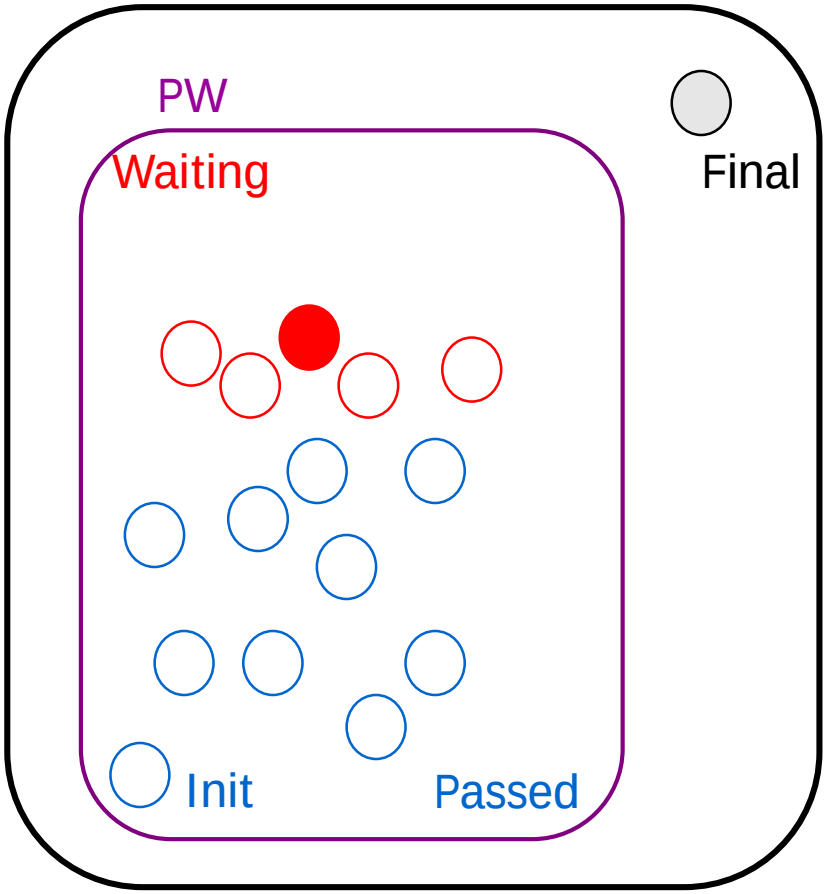
INITIAL Passed := Ø;
Waiting := $\{(n_0,Z_0)\}$

REPEAT
pick (n,Z) in Waiting
if (n,Z) = Final return **true**
for all (n,Z)→(n',Z'):
if for some (n',Z'') Z'⊆ Z'' continue
else add (n',Z') to Waiting
move (n,Z) to Passed

UNTIL Waiting = Ø
return **false**

Forward Reachability Algorithm

Init -> Final ?



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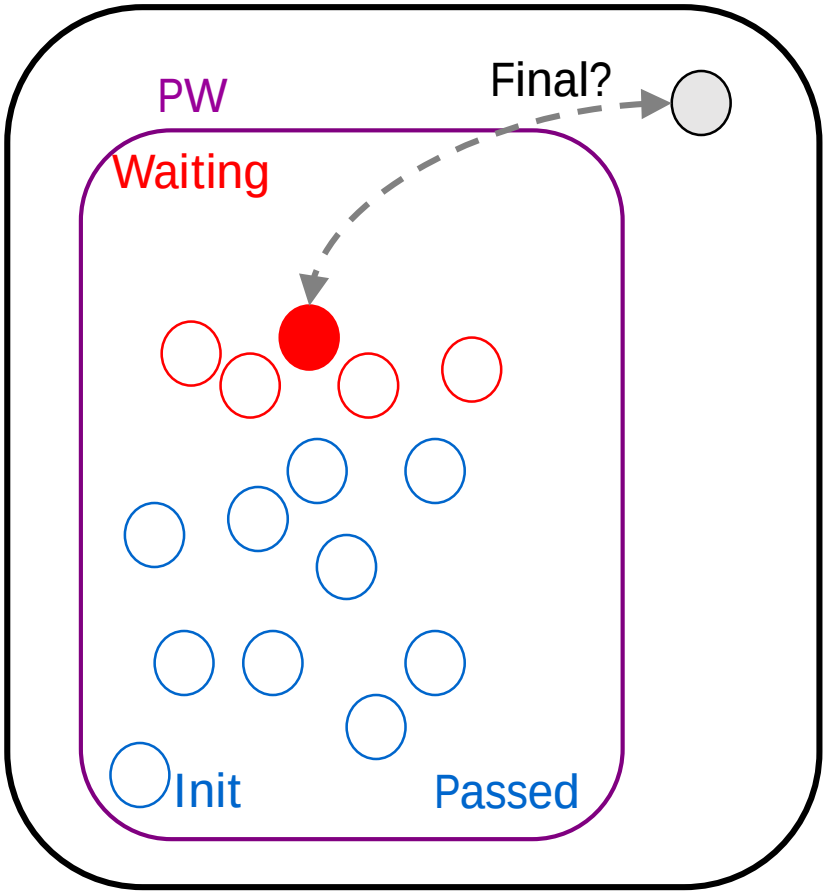

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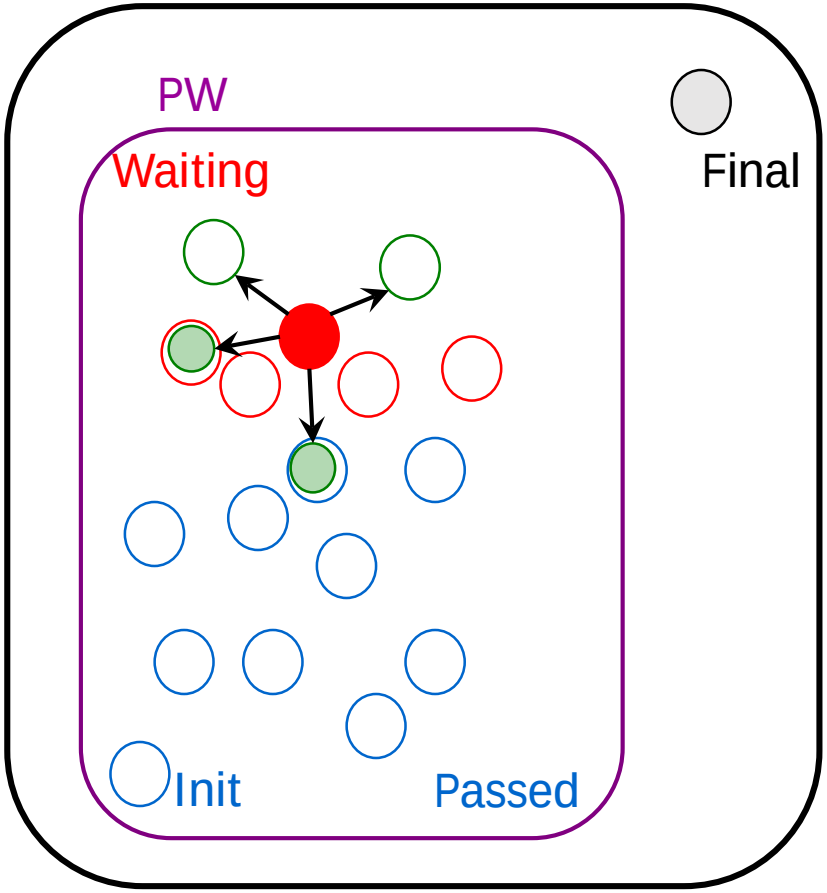

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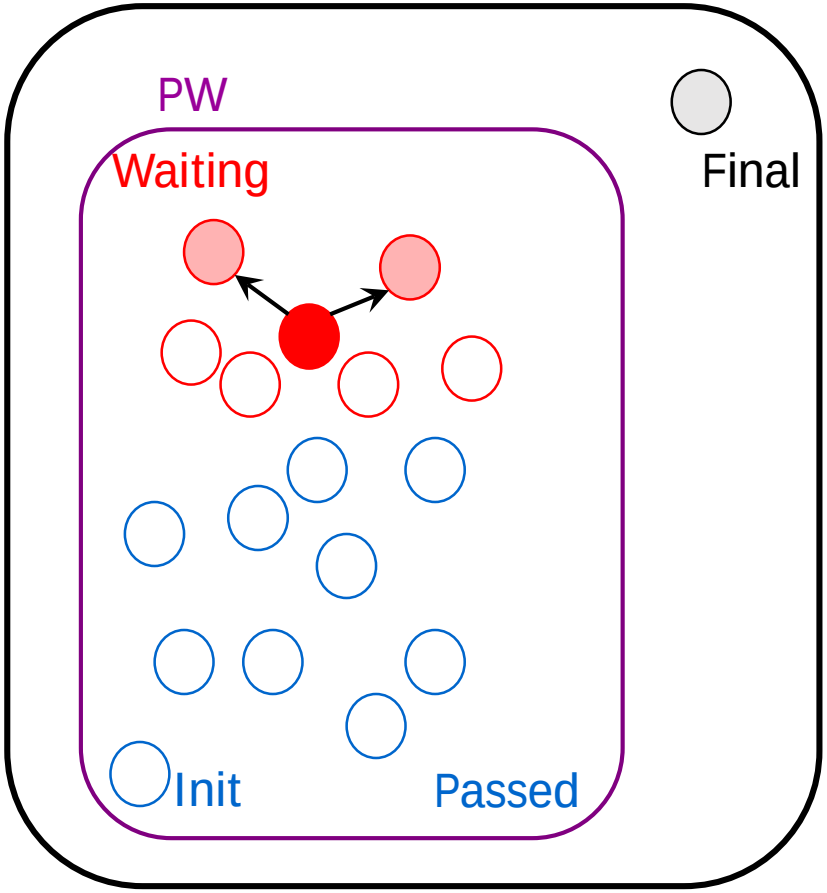

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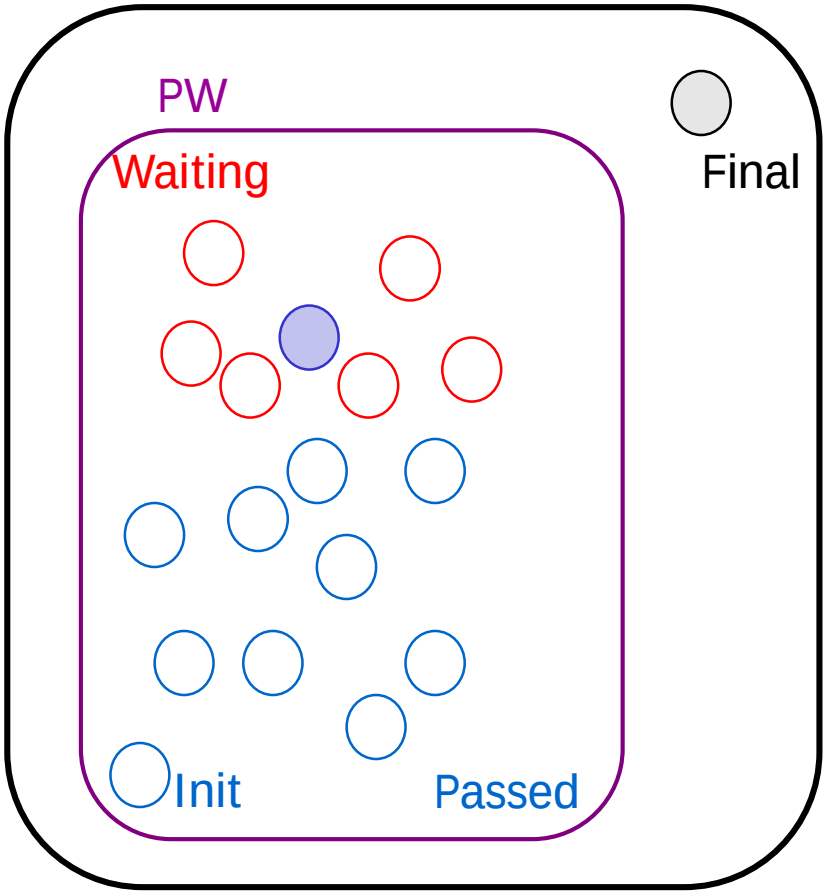

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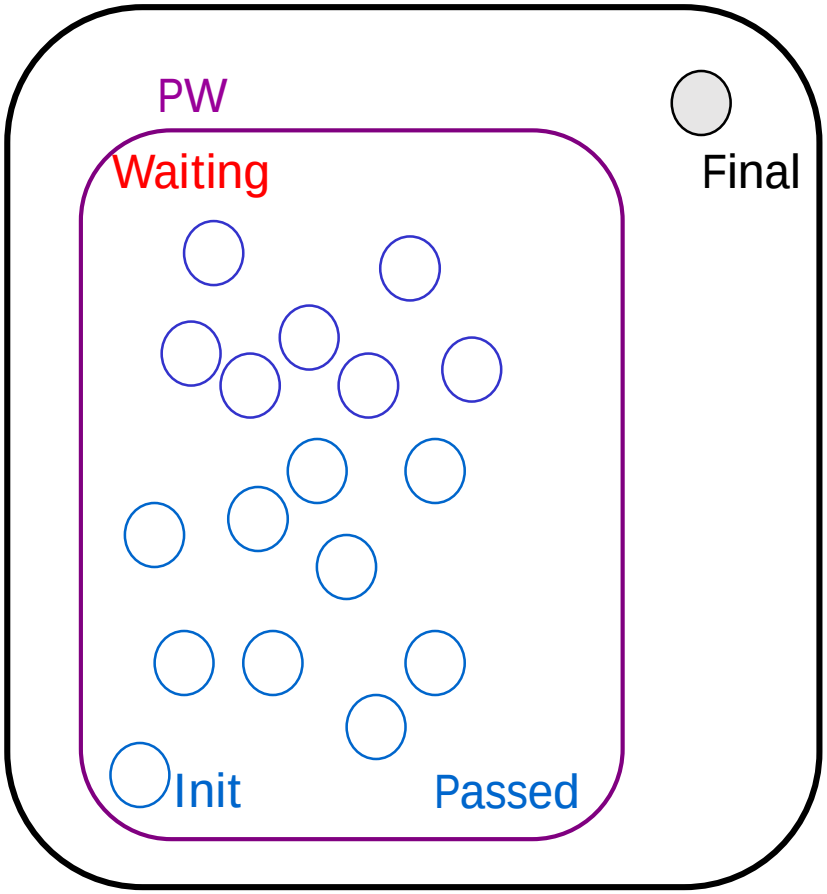

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Specification (Query) Language

UPPAAL Property Specification Language

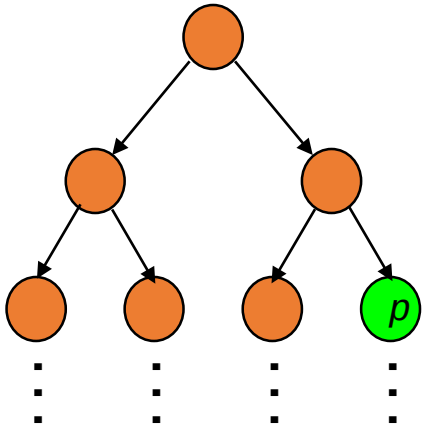
- **A[] p** *always*
- **A<> p** *inevitable*
- **E<> p** *Possible*
- **E[] p** *potentially always*
- **P --> q** *leads-to*


```
p::= a.l | gd | gc | p and p |
     p or p | not p | p imply p |
     ( p ) | deadlock(only for A[],E<>)
```

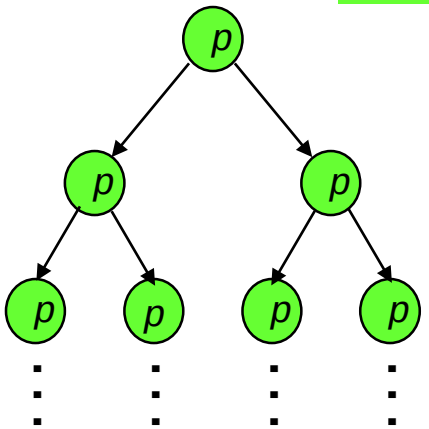
```
A[] (mc1.finished and mc2.finished) imply (accountA+accountB==200)
```

Uppaal “Computation Tree Logic”

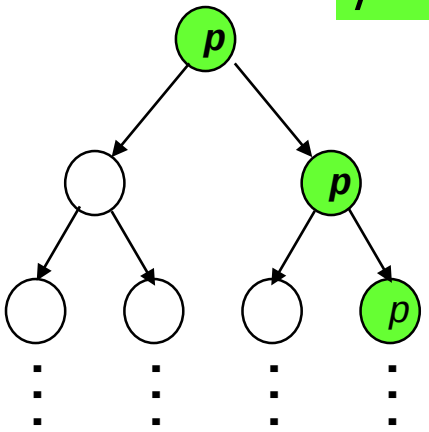
E<> *p* **Possible**



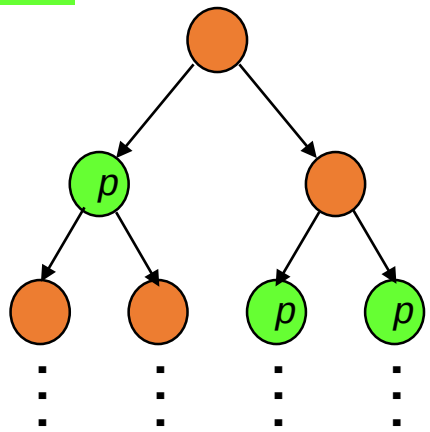
A[] *p* **always**



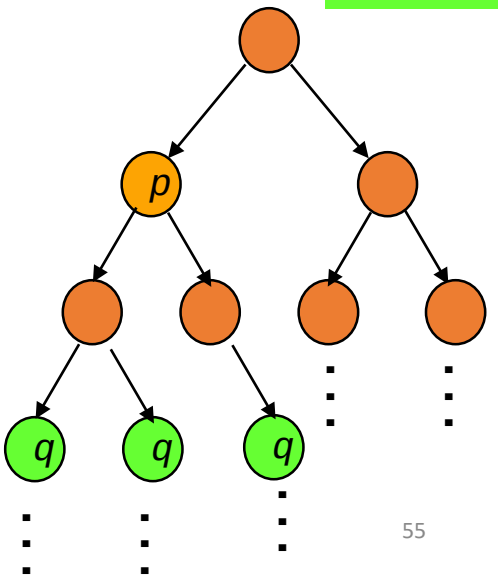
E[] *p* **potentially always**



A<> *p* **inevitable**



p --> *q* **leads-to**



Logical Specifications

- Validation Properties
  - Possibly: E<> *p*
- Safety Properties
  - Invariant: A[] *p*
  - Possibly Inv.: E[] *P*
- Liveness Properties
  - Eventually: A<> *p*
  - Leads_to: *p* --> *p*
- Bounded Liveness
  - Leads to within: $p \dashrightarrow_{\leq t} q$

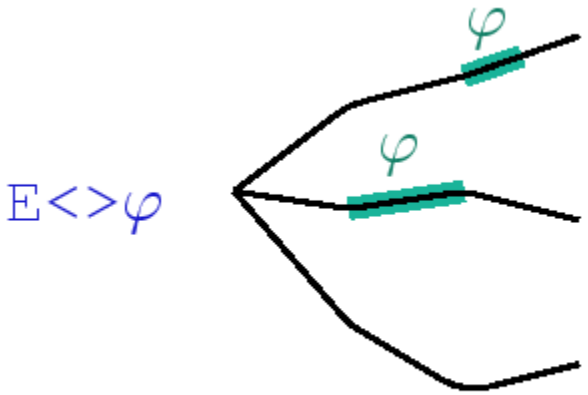
The expressions *p* and *q*

- must be type safe, side effect free, and evaluate to a boolean.

- only references to integer variables, constants, clocks, and locations are allowed (and arrays of these).

Logical Specifications

- Validation Properties
  - Possibly: E<> φ
- Safety Properties
  - Invariant: A[] φ
  - Pos. Inv.: E[] φ
- Liveness Properties
  - Eventually: A<> φ
  - Leadsto: φ --> ψ
- Bounded Liveness
  - Leads to within: φ --> $_{\leq t}$ ψ



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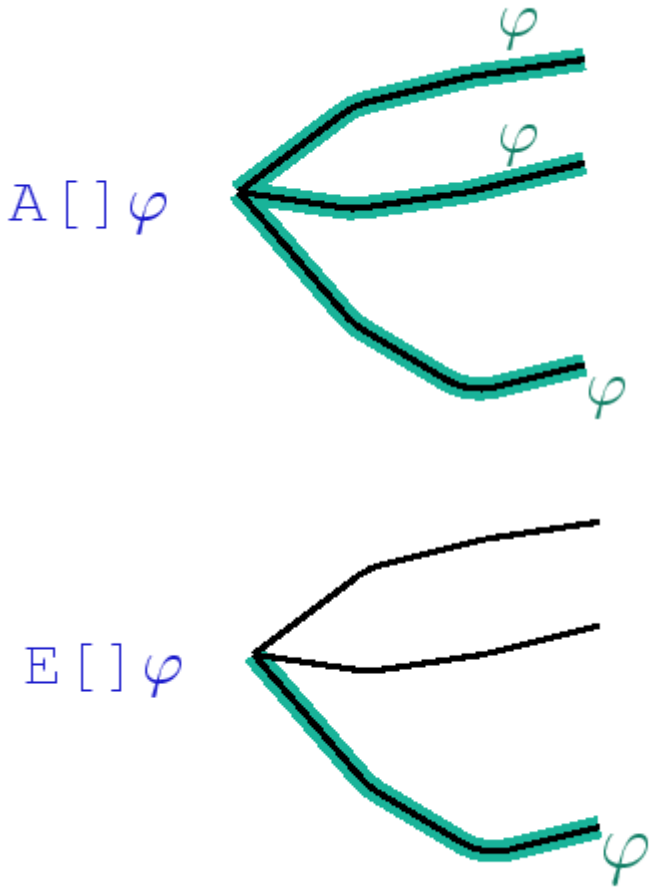

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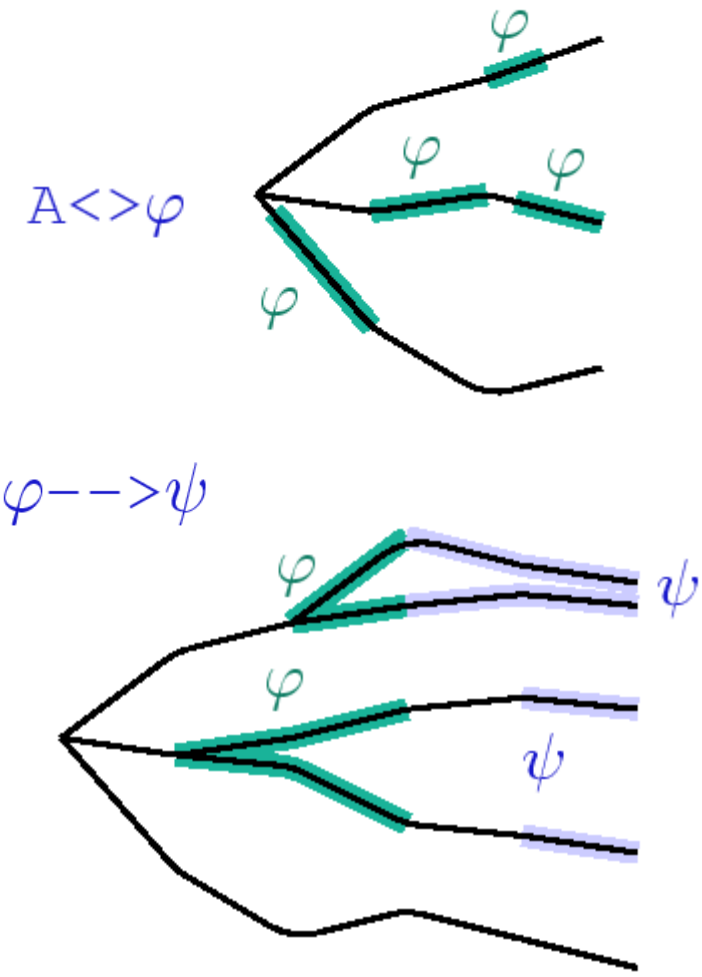

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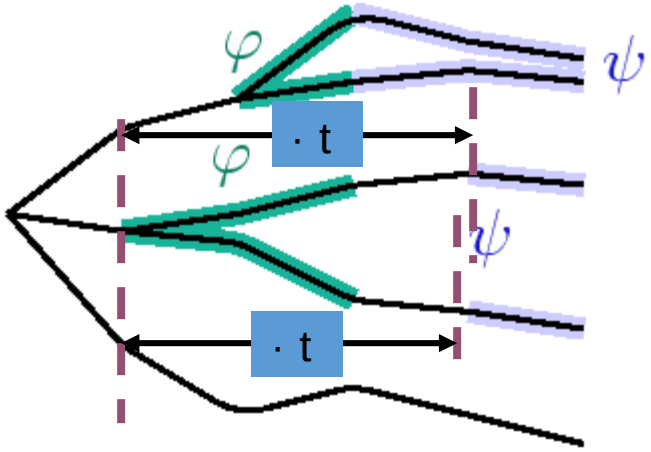

Jug Example

- Safety: Never overflow.
  - A[] forall(i:id_t) level[i] <= capa[i]

- Validation/Reachability: How to get 1 unit.
  - E<> exists(i:id_t) level[i] == 1

Train-Gate Crossing

- Safety: One train on crossing at a time.
  - A[] forall (i : id_t) forall (j : id_t)
    Train(i).Cross && Train(j).Cross imply i == j
- Liveness: Approaching trains eventually cross.
  - Train(0).Appr --> Train(0).Cross
  - Train(1).Appr --> Train(1).Cross
  - …
- No deadlock.
  - A[] not deadlock