

# Kahendpuu (Binary tree)

Programmeerimise süvendatud  
algkursus (ITI0140)

2015

# Teemad

- aheljärjend (*Linked list*)
- Puu (*Tree*)
- Kahendpuu (*Binary tree*)
- Kahendpuu tüübid

# Terms (*mõisted*)

- Node (*sõlm*)
- Root (*juur*)
- Parent node (*vanem sõlm*)
- Child node (*laps-sõlm*)
- Sibling (*õved*)
- Path (*tee*)

# Embedded reference

- Attributes which refer to other objects are called **embedded references**
- Objects are said to be **linked** if an instance of one object stores a reference to another object
- Special case of linkage is when an object contains a link to an object of the **same class**

# Linked list

- Linked list is made up of **nodes**
- Each node contains a **reference to the next node** in the list
- In addition, each node contains a unit of data called the **cargo**

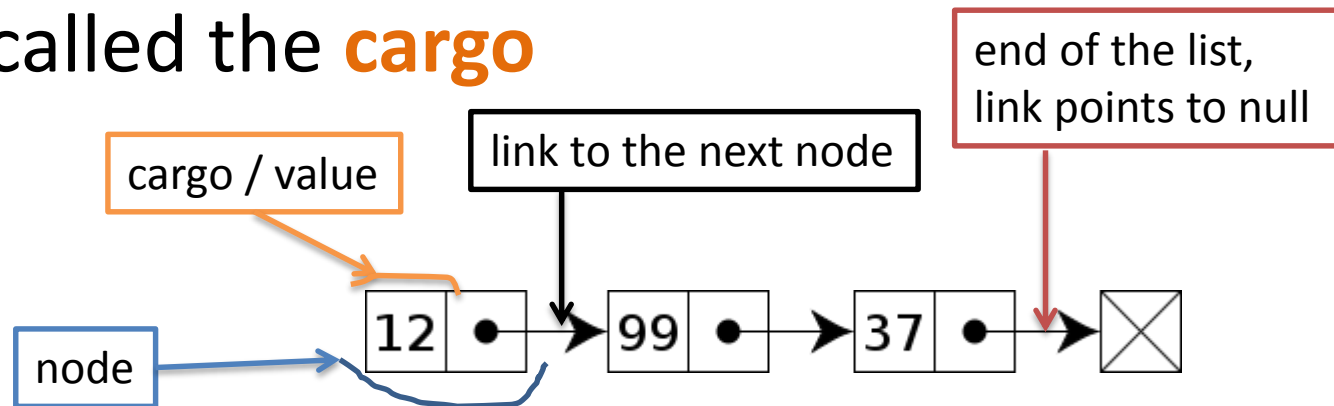


Image: <http://en.wikipedia.org/wiki/File:Singly-linked-list.svg>

# Linked list

- Node:

```
class Node:
    def __init__(self, cargo=None, next=None):
        self.cargo = cargo
        self.next = next

    def __str__(self):
        return str(self.cargo)
```

- Example:

```
node1 = Node(1)
node2 = Node(2)
node3 = Node(3)
```

```
node1.next = node2
node2.next = node3
```

# Linked list

- Adding node

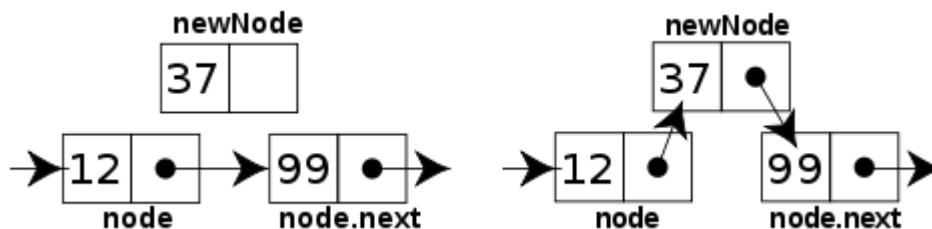


Image: <http://en.wikipedia.org/wiki/File:CPT-LinkedLists-addingnode.svg>

- Removing node

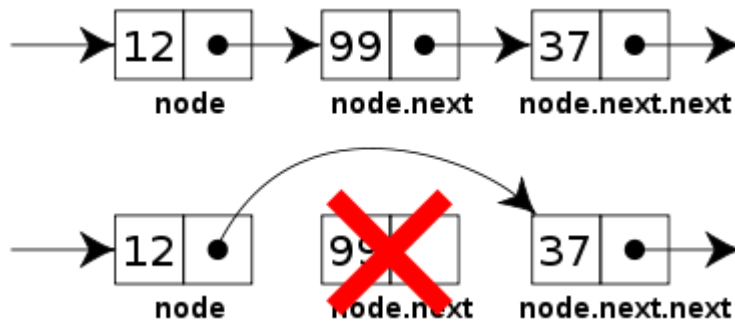


Image: <http://en.wikipedia.org/wiki/File:CPT-LinkedLists-deletingnode.svg>

# Tree

- Like linked lists, trees are made up of nodes
- The top of the tree is called the **root**
- Other nodes are called **branches**
- Nodes with null reference are called **leaves**
  
- The top node is called **parent** and the nodes it refers to are its **children**
- Nodes with the same parent are called **siblings**
- All the nodes which are the same distance from the root comprise a **level** of the tree

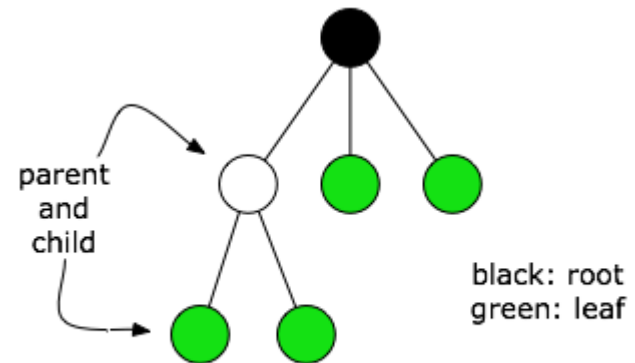
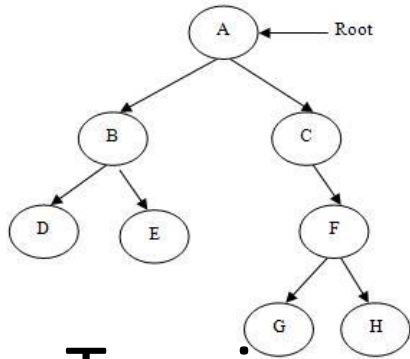


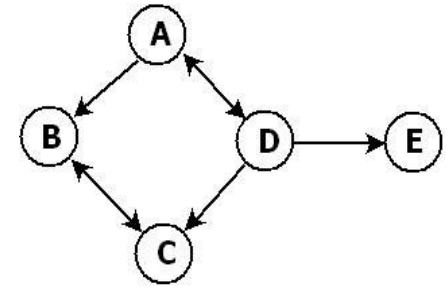
Figure: tree data structure

Image: <http://xlinux.nist.gov/dads/HTML/tree.html>





# Tree vs graph



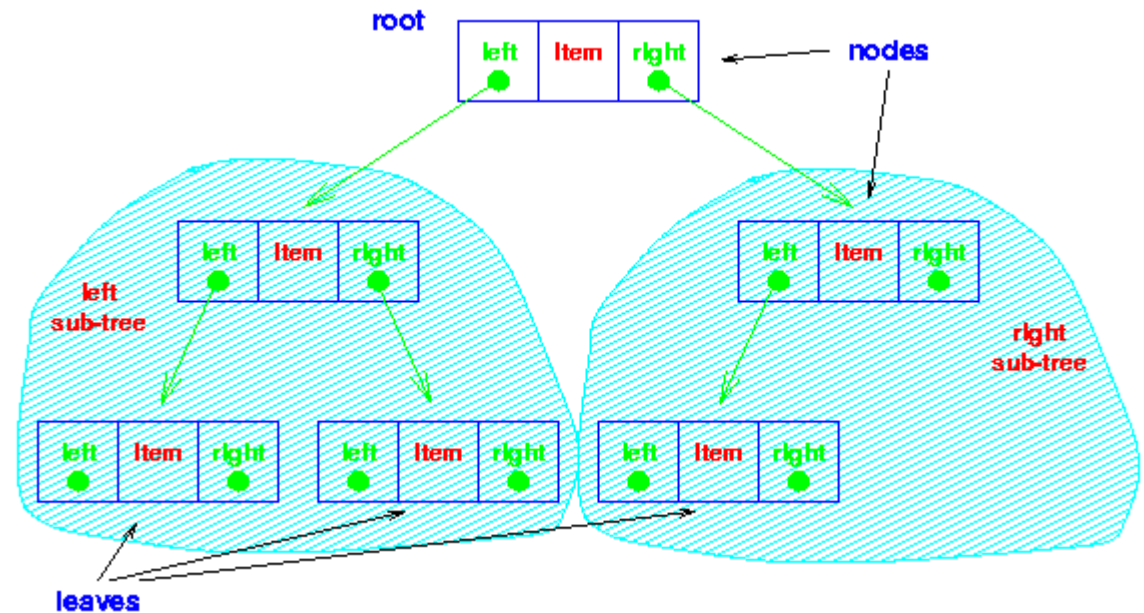
- Tree is a special form of graph, **minimally connected graph** having only one path between any two vertices
- Tree has **no loops**
- Tree has exactly **one root node**
- In tree, every **child** node has only **one parent**
- Tree has always **n-1 edges**

Comparison: <http://freefeast.info/difference-between/difference-between-trees-and-graphs-trees-vs-graphs/>

# Binary tree

- Tree, where each node contains a reference to two other nodes (possibly None)
- References are referred to as the left and right sub-trees

- Binary tree consists of:
  - a root node
  - left and right sub-trees
- Both the sub-trees are themselves binary trees



# Binary tree

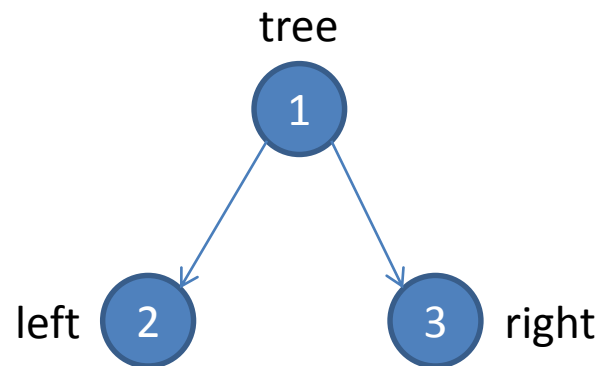
- Node:

```
class Tree:
    def __init__(self, cargo, left=None, right=None):
        self.cargo = cargo
        self.left = left
        self.right = right

    def __str__(self):
        return str(self.cargo)
```

- Example:

```
left = Tree(2)
right = Tree(3)
tree = Tree(1, left, right)
```



# Sorted binary tree

## Binary search tree

- **Binary search tree (BST)**, sometimes also called sorted or ordered binary tree:
  - the **left sub-tree** of a node contains only nodes with **values less than** node's key
  - the **right sub-tree** of a node contains only nodes with **values greater than** node's key
  - the left and right sub-trees have to be binary search trees
  - there must be **no duplicate nodes**

# Searching BST

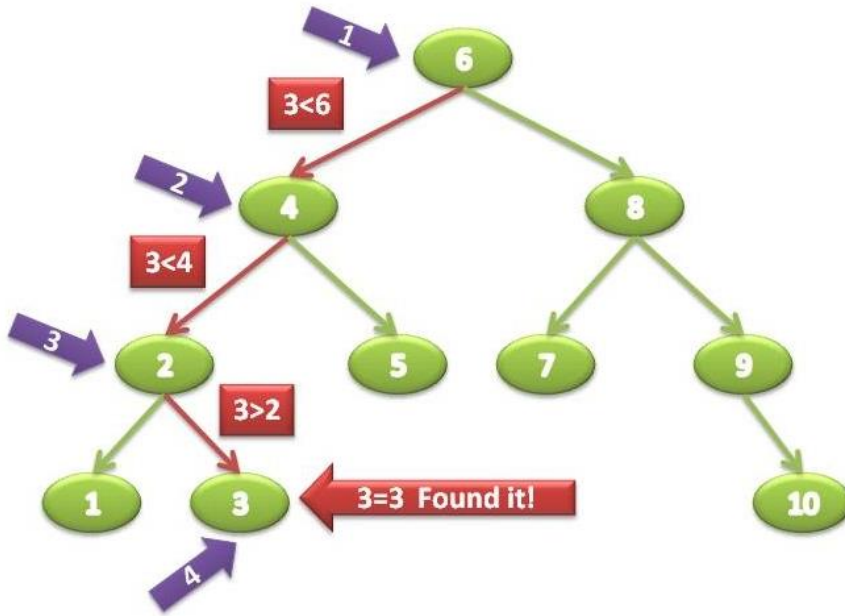


Image: <http://encrypt3d.wordpress.com/2010/09/25/how-to-search-in-a-binary-search-tree/>

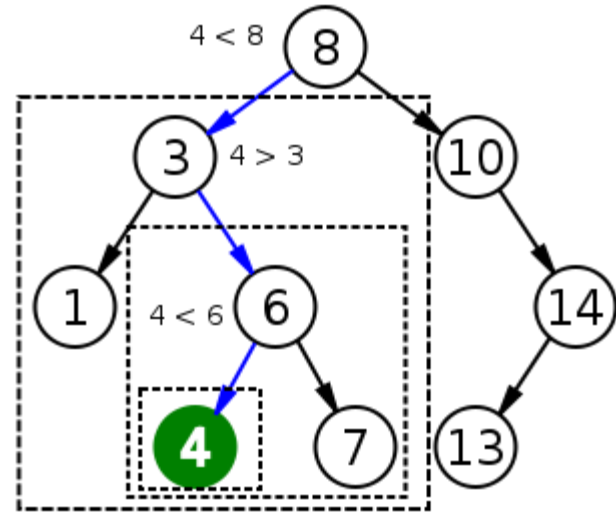
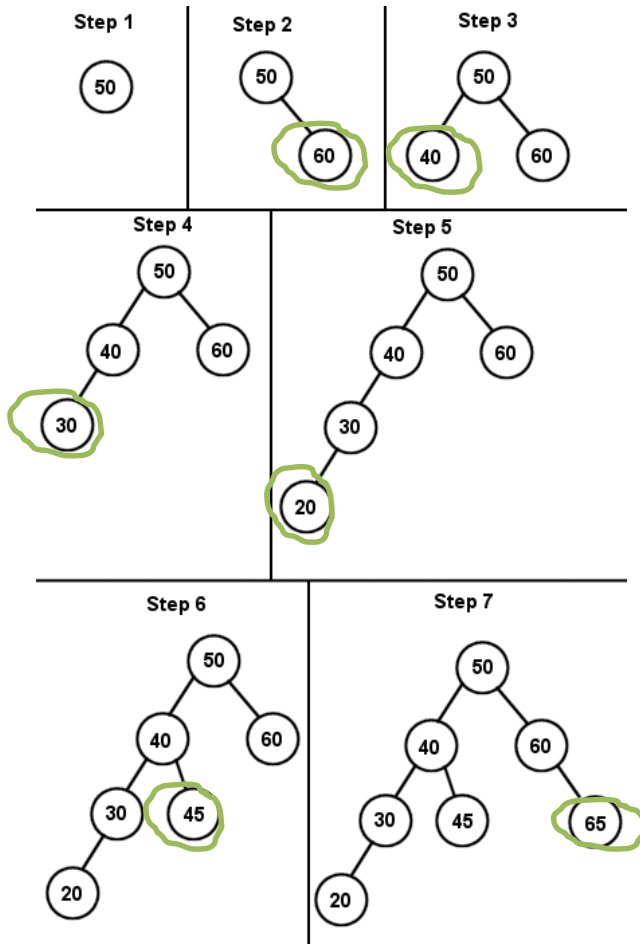
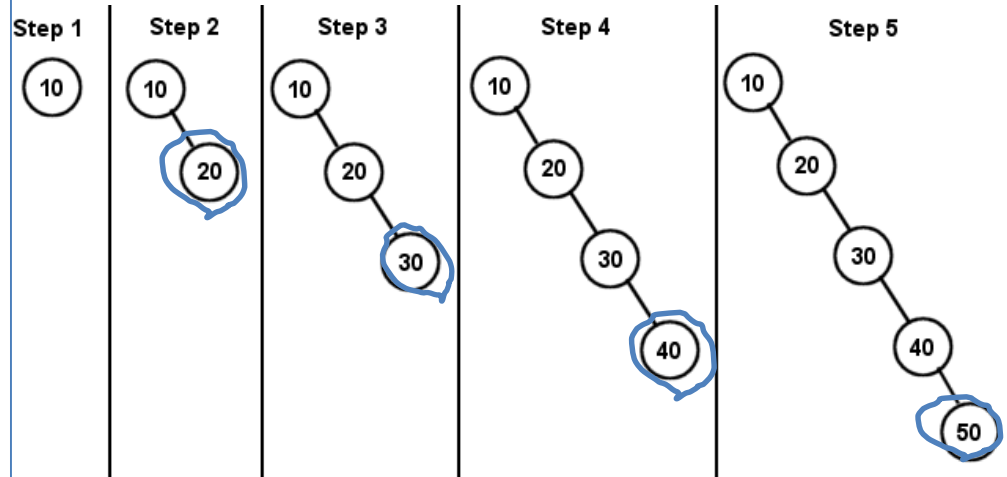


Image: [http://commons.wikimedia.org/wiki/File:Binary\\_search\\_tree\\_search\\_4.svg](http://commons.wikimedia.org/wiki/File:Binary_search_tree_search_4.svg)

# Inserting to BST



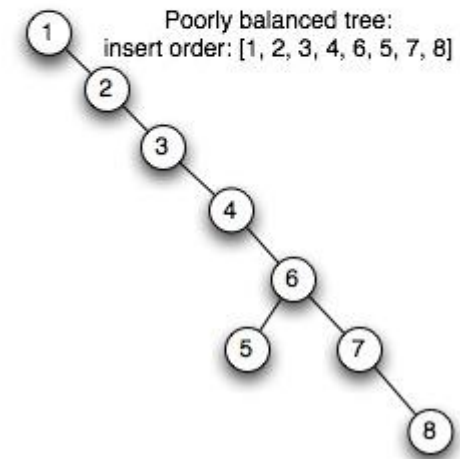
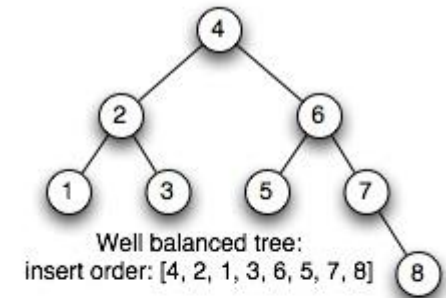
Worst case:



Images: <http://archive.gamedev.net/archive/reference/articles/article1433.html>

# Balanced binary tree

- A **balanced binary tree** is commonly defined as a binary tree in which the depth of the left and right sub-trees of every node differ by 1 or less
- Balanced binary tree has a predictable depth
- Lookup, insertion, deletion completed in  $O(\log n)$  time for  $n$  node tree



# Additional info

- [http://en.wikipedia.org/wiki/Binary\\_tree](http://en.wikipedia.org/wiki/Binary_tree)
- <http://archive.gamedev.net/archive/reference/articles/article1433.html>
- <http://algs4.cs.princeton.edu/32bst/>
- <http://encrypt3d.wordpress.com/category/data-structures/binary-tree/>
- [http://en.wikipedia.org/wiki/Self-balancing\\_binary\\_search\\_tree](http://en.wikipedia.org/wiki/Self-balancing_binary_search_tree)



# Ülesanne

- vt [ained.ttu.ee](http://ained.ttu.ee)