1. Show that $\mathbb{Z}$ fails to be a group under multiplication.

2. Show that $\mathbb{Z} \setminus \{0\}$ fails to be a group under multiplication.

3. Show that $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ is a group under addition operation defined by $(a, b) + (c, d) = (a + c, b + d)$.

4. What is the order of group $U(12)$ (the group of units)?

5. Is $\{0, 2\}$ a subgroup of $\mathbb{Z}_4$?

6. What are the subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2$?

7. Show that $\{-1, 1, i, -i\}$ is a subgroup of $(\mathbb{C} \setminus \{0\}, \times)$.

8. Is $\mathbb{Z}$ a cyclic group?

9. Show that $\mathbb{Z}_6$ is generated by both 1 and 5.

10. Is $3\mathbb{Z}$ a cyclic subgroup of $\mathbb{Z}$?

11. What is the order of 4 in $\mathbb{Z}_6$?

12. What is the order of 2 in $\mathbb{Z}_5$? Does 2 generate $\mathbb{Z}_5$?

13. What is the order of 2 in $U(5)$?

14. What is the order of 5 in $U(12)$?

15. What is the order of $-i \in \mathbb{C} \setminus \{0\}$?

16. What is the group structure of $U(9)$? Is $U(9)$ a cyclic group?

17. What is the group structure of $U(8)$? Is $U(8)$ a cyclic group?

18. If $a^{24} = e$ in group G, what are possible orders of a?

19. Suppose G is a finite group with an element g with order 5, and and an element h of order 7. What are possible orders of G?

20. Show that $U(8)$ and $\mathbb{Z}_4$ have different group structures.

21. Show that $U(5)$ and $U(10)$ have the same group structure, but not $U(12)$.

22. Do groups $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ have the same group structure?

23. Do groups $\mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2$ and $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ have the same group structure?