1. Apply the Euclidean algorithm and calculate

$$\gcd(26, 9) \qquad\qquad \gcd(81, 18)$$

Solution.

$$\gcd(26, 9) = \gcd(9, 8) = \gcd(8, 1) = \gcd(1, 0) = 1$$
$$\gcd(81, 18) = \gcd(18, 9) = \gcd(9, 0) = 9$$

2. Express the following pairs of numbers in the form of Bezout identity

$$\alpha a + \beta b = \gcd(a, b) \ .$$

$$(60, 12) \qquad\qquad (12, 18) \qquad\qquad (26, 9)$$

Solution.

60	12	a	b
0	12	a-5b	b

12	18	a	b
12	6	a	b-a
0	6	a-2(b-a)=3a-2b	b-a

26	9	a	b
8	9	a-2b	b
8	1	a-2b	b-(a-2b)=3b-a
0	1	a-2b-8(3b-a) = 9a-26b	3b-a

The Bezout identities are:

$$0 \cdot 60 + 1 \cdot 12 = \gcd(60, 12) \ ,$$
$$-1 \cdot 12 + 1 \cdot 18 = \gcd(12, 18) \ ,$$
$$-1 \cdot 26 + 3 \cdot 9 = \gcd(26, 9) \ .$$

3. Provide prime factorization of the following integers:

(a) 64 $\qquad$ (b) 120

(c) 375 $\qquad$ (d) 47

Solution.

(a) $64 = 2^6$ $\qquad$ (b) $120 = 2^3 \cdot 3 \cdot 5$

(c) $375 = 15 \cdot 25 = 3 \cdot 5^3$ $\qquad$ (d) 47

4. Calculate the Euler's function $\varphi(n)$ for the following n:

(a) 64 $\qquad$ (b) 120

(c) 375 $\qquad$ (d) 47

Solution.

$(a)\quad \varphi(64) = \varphi(2^6) = 64 \cdot \left(1 - \frac{1}{2}\right) = 32$

$(b)\quad \varphi(120) = \varphi(2^3 \cdot 3 \cdot 5) = 120 \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{5}\right) = 120 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4}{5} = 32$

$(c)\quad \varphi(375) = \varphi(3 \cdot 5^3) = 375 \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{5}\right) = 375 \cdot \frac{2}{3} \cdot \frac{4}{5} = 200$

$(d)\quad \phi(47) = 47 - 1 = 46$