1 Theory

Indices of letters:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Measure of Roughness (**MR**) is a measure how much a distribution differs from a uniform distribution.

$$\mathbf{MR} = \sum_i \left(p_i - \frac{1}{26}\right)^2 = \sum_i p_i^2 - 2\frac{1}{26}\underbrace{\sum_i p_i}_{=1} + \underbrace{\sum_i \left(\frac{1}{26}\right)^2}_{=26\cdot\frac{1}{26^2}} = \sum_i p_i^2 - \frac{1}{26} \approx \sum_i p_i^2 - 0.038 \ .$$

$\sum_i p_i^2$ os the probability that any two letters randomly selected from a distribution will be the same. Index of coincidence **IC** is an approximation to $\sum_i p_i^2$. In a set of N elements, element a can form $\binom{f_a}{2} = \frac{f_a \cdot (f_a - 1)}{2}$ pairs, where f_a is the number of times letter a appears in the set. The total number of possible pairs in a set of N letters is $\binom{N}{2} = \frac{N \cdot (N-1)}{2}$. The probability that two randomly selected letters will be "A"-s is

$$\frac{\binom{f_A}{2}}{\binom{N}{2}} = \frac{f_a \cdot (f_a - 1)}{N \cdot (N-1)} \ ,$$

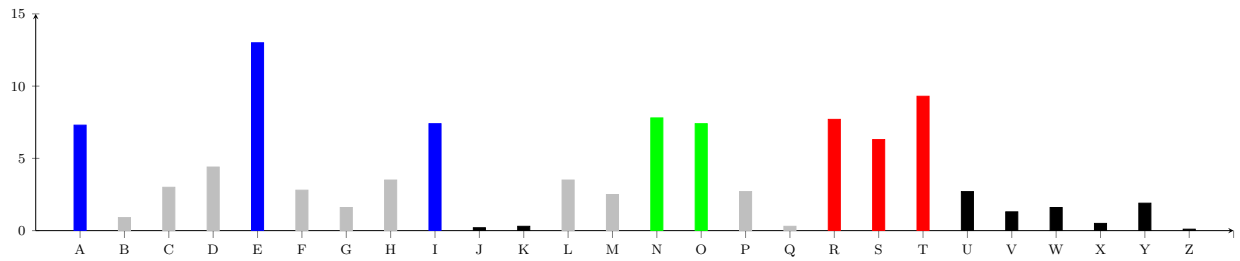
and the index of coincidence is just the sum over all possible letters:

$$\mathbf{IC}(\mathrm{Y}) = \sum_i \frac{f_i \cdot (f_i - 1)}{N \cdot (N-1)} \ .$$

I.C. approximates the probability that any two letters randomly sampled from a distribution will be the same. Since IC approximates $\sum_i p_i^2$, it has the same range of variation 0.038 to 0.066, which corresponds to the sum of squares of the characteristic frequencies of English characters. The lower bound corresponds to a uniform distribution, and the upper bound correponds to monoalphabeticity.

On average, in a 1000 letter long sample of English text, the letters are distributed as follows:

A 73	*B* 9	*C* 30	*D* 44	*E* 130	*F* 28
G 16	*H* 35	*I* 74	*J* 2	*K* 3	*L* 35
M 25	*N* 78	*O* 74	*P* 27	*Q* 3	*R* 77
S 63	*T* 93	*U* 27	*V* 13	*W* 16	*X* 5
Y 19	*Z* 1				



The same picture would result from the examination of any reasonably long plain language text. Relative frequencies may vary slightly, but the basic facts remain the same:

- Evenly spaced vowels `A E I` with high frequency are evenly spaced 4 letters apart.
- Letter `E` is the most frequent of all the letters
- Consecutive part `N,O` have high frequency
- Consecutive triplet `R,S,T` has high frequency
- The pair `J,K` has low frequency
- The string `U,V,W,X,Y,Z` has low frequency.

2 Tasks

1. An additive cipher maps plaintext G to ciphertext X. What is the encryption key? Which decryption key will allow to reconstruct the plaintext?
2. We know that a ciphertext was produced by a shift cipher, and that the encryption key was 17. What is the decryption key?
3. We know that the plaintext word `THE` is encrypted by an affine cipher into trigam `NHM`. What is the encryption key? What is the decryption key?
4. A ciphertext obtained by an affine cipher with key $(3, 17)$. Which key will you use to decrypt it?
5. What is the I.C. of the ciphertext `EPYEPOPDZSZUFPO`?
6. Encrypt the word `MORNING` using a shift cipher with key 11.
7. Encrypt the word `SYMBOL` using an affine cipher with key $(3, 2)$.
8. Encrypt the word `PARADOX` using a Vigenère cipher with key `YESTERDAY`.
9. Decipher the following messages:

   ```
   ESPCP TD L ETOP TY ESP LQQLTCD ZQ XPY HSTNS ELVPY LE ESP QWZZO WPLOD ZY EZ
   QZCEFYP
   ```

   ```
   SV SQ VNY OWMMWR KWRTYL WD EHH MYR NWK EMWRI QW MERU MSHHSWRQ WD DEOYQ VNYLY
   QNWAHT
   ```

   ```
   BDSTGC WXHIDGN AXZT P STPU BPC PCHLTGH FJTHIXDCH CD DCT PHZTS
   ```

   ```
   JDI HVANGNKFKJS JDGJ EI MGS PGKF KT G EAVJDS UGQCI KC TAJ CQPPKUKITJ RQCJKPKUGJKAT
   PAV AQV VIPQCKTW JA CQHHAVJ KJ
   ```