

1. Show that \mathbb{Z} fails to be a group under multiplication.
2. Show that $\mathbb{Z} \setminus \{0\}$ fails to be a group under multiplication.
3. Show that $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ is a group under addition operation defined by $(a, b) + (c, d) = (a + c, b + d)$.
4. What is the order of group $U(12)$ (the group of units)?
5. Is $\{0, 2\}$ a subgroup of \mathbb{Z}_4 ?
6. What are the subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2$?
7. Show that $\{-1, 1, i, -i\}$ is a subgroup of $(\mathbb{C} \setminus \{0\}, \times)$.
8. Is \mathbb{Z} a cyclic group?
9. Show that \mathbb{Z}_6 is generated by both 1 and 5.
10. Is $3\mathbb{Z}$ a cyclic subgroup of \mathbb{Z} ?
11. What is the order of 4 in \mathbb{Z}_6 ?
12. What is the order of 2 in \mathbb{Z}_5 ? Does 2 generate \mathbb{Z}_5 ?
13. What is the order of 2 in $U(5)$?
14. What is the order of 5 in $U(12)$?
15. What is the order of $-i \in \mathbb{Z} \setminus \{0\}$?
16. What is the group structure of $U(9)$? Is $U(9)$ a cyclic group?
17. What is the group structure of $U(8)$? Is $U(8)$ a cyclic group?
18. If $a^{24} = e$ in group G , what are possible orders of a ?
19. Suppose G is a finite group with an element g with order 5, and an element h of order 7. What are possible orders of G ?
20. Show that $U(8)$ and \mathbb{Z}_4 have different group structures.
21. Show that $U(5)$ and $U(10)$ have the same group structure, but not $U(12)$.
22. Do groups \mathbb{Z}_4 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ have the same group structure?
23. Do groups $\mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2$ and $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ have the same group structure?