Homework 1 – Sets

Exercise 1. Given the two subsets

$$A = \{x \in \mathbb{R} : 0 < x \leqslant 3\} \ ,$$
$$B = \{x \in \mathbb{R} : 2 \leqslant x < 4\} \ ,$$

of the universal set $\mathbb{R}$, define the following sets: $A \cap B$; $\quad A \cup B$; $\quad A \setminus B$; $\quad A'$.

Exercise 2. Suppose that

$$A = \{x \in \mathbb{N} : x \text{ is even}\} \ ,$$
$$B = \{x \in \mathbb{N} : x \text{ is prime}\} \ ,$$
$$C = \{x \in \mathbb{N} : x \text{ is a multiple of } 5\} \ .$$

Describe each of the following sets.

(a) $A \cap B$ $\qquad$ (b) $B \cap C$

(c) $A \cup B$ $\qquad$ (d) $A \cap (B \cup C)$

Exercise 3. If $A = \{a, b, c\}, B = \{1, 2, 3\}, C = \{x\}$, and $D = \emptyset$, list all of the elements in each of the following sets.

(a) $A \times B$ $\qquad$ (b) $B \times A$

(c) $A \times B \times C$ $\qquad$ (d) $A \times D$

Exercise 4. Let A be a set. Show that $A \cap A = A$.

Exercise 5. Let A be a set. Show that $A \cup \emptyset = A$.

Exercise 6. Let A, B, C be sets. Show that $A \cup (B \cup C) = (A \cup B) \cup C$.

Exercise 7. Let A, B be sets. Show that $A \cup B = B \cup A$.

Exercise 8. Let A, B, C be sets. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Exercise 9. Let A, B be sets. Show that $(A \cup B)' = A' \cap B'$.

Exercise 10. Let A, B, C be sets. Show that $A \cup B = (A \cap B) \cup (A \setminus B) \cup (B \setminus A)$.