Exercise 15

Show that

```
⊢ {M≥1}
 BEGIN
  X:=0;
  FOR N:=1 UNTIL M DO X:=X+N
 END
  {X=(M×(M+1)) DIV 2}
```

Exercise 21

Show that

```
⊢ {M≥0}
 BEGIN
   X:=0;
   FOR N:=1 UNTIL M DO X:=X+N
 END
  {X=(M×(M+1)) DIV 2}
```

□

Exercise 22

Deduce:

```
⊢ {S = (x×y)-(X×Y)}
 WHILE ¬ODD(X) DO
   BEGIN Y:=2×Y; X:=X DIV 2 END
  {S = (x×y)-(X×Y) ∧ ODD(X)}
```

□

Exercise 25

Prove the following invariant property.

```
⊢ {S = (x-X)×y ∧ Y=y}
 BEGIN
   VAR R;
   R:=0;
   WHILE ¬(R=Y) DO
     BEGIN S:=S+1; R:=R+1 END;
   X:=X-1
 END
  {S = (x-X)×y}
```

Hint: Show that `S = (x-X)×y + R` is an invariant for `S:=S+1; R:=R+1`. □

Exercise 28

Prove that the command

```
BEGIN
  Z:=0;
  WHILE ¬(X=0) DO
    BEGIN
      IF ODD(X) THEN Z:=Z+Y;
      Y:=Y×2;
      X:=X DIV 2
    END
END
```

computes the product of the initial values of `X` and `Y` and leaves the result in `Z`. □

Invariant for Exercise 15: $\boxed{R \equiv X = N * (N - 1) \text{ DIV } 2}$