Formalizing uncertain knowledge

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Overview of the area

- Not related to agent/person:
  - Discrete:
    - Multi-valued logic
    - Default logic
    - Other model-based systems
  - Non-discrete:
    - Probabilistic (several types!)
    - Fuzzy (possibilistic)
- Related to agent/person (not covered in this presentation):
  - Logics of knowledge and belief
  - Other “meta”-systems

Why different uncertainties 1

- Real-life rules have **exceptions**: not all birds fly, not all things fall down, etc. We **do not have statistics** on how often birds do not fly etc.
- We can numerically estimate the **probability** that on 27 oct the temperature in Tallinn falls below 0: we have statistics.
- People have **opinions which can be statistically measured**: how high percentage of people asked will say that the bar Shvips is a good place for drinks at night? For a quick snack during day? A good place to look football?

Why different uncertainties 2

- People have **opinions of different strength:** how confident are you that Trump is the U.S. president? That Oswald had co-conspirators? That aliens control everything?
- Ordinary words have somewhat measureable meaning graphs: is the man of height 2 meters **tall**? Is the man of height 1.9 meters tall? Of 1.8? Of 1.7?
- Different people have different belief and knowledge: **who knows of believes** what?

Rules with exceptions

- Since rules, logic.
- A few different logics for rules with exceptions: none of them widely used.

- First, the problem: in normal logic there are no „exceptions“. If birds fly, but Tweety is a bird and does not fly, **we have a contradiction** and all queries essentially fail.

Default Reasoning

The problem: in FOL, universally-quantified rules cannot have exceptions

- $\forall x\ bird(x) \rightarrow can\_fly(x)$
- bird(tweety)
- $bird(opus) \wedge \neg can\_fly(opus)$

as soon as you assert something contradictory, the knowledge base becomes inconsistent

- no models satisfy can_fly(opus) and $\neg$can_fly(opus)
- arbitrary conclusions can be drawn from an inconsistent knowledge base

could add qualifying antecedents, but you have to know/anticipate all possible exceptions

- $\forall x\ bird(x) \wedge \neg penguin(x) \wedge \neg dead(x) \wedge \neg in\_cage(x) \wedge \ldots \rightarrow can\_fly(x)$

Non-montonicity

FOL is monotonic

whenever you add something to a knowledge base, everything that was previously entailed is still true

if KB $\models \alpha$ then KB$\wedge\beta \models \alpha$

why? because adding β restricts the models to a subset, but they all still satisfy α

Non-monotonic logics (alternatives to FOL)

default logic

circumscription

...

Default Logic: syntax

Prerequisite : Justification / Conclusion

Bird(X) : Flies(X) / Flies(X)

Bird(X) & not derivable (-Flies(X)) => Flies(X)

read as: if X is a bird *and it is not inconsistent to believe that* X flies, then conclude that X flies

thus if

KB={ bird(X) : flies(X) / flies(X),

bird(tweety),

nonliving(X) : -flies(X) / -flies(X)

nonliving(opus)

bird(opus)}

then KB $\models$ flies(tweety) but not flies(opus)

Default Logic: semantics

define "minimal" models as models of the FOL subset (non-default sentences)

m_1={bird(tweety)=T,bird(opus)=T,flies(opus)=F}

define "extensions" of models by an operator that adds a fact from a default rule one at a time, e.g. apply to tweety...

m_2={bird(tweety)=T,bird(opus)=T,flies(opus)=F,flies(tweety)=T}

define "fixed points" as models that result from iteratively applying this operator until no more conclusions can be drawn

entailments consists of things true in some extension

Nixon diamond example:

Republican(Nixon)$\wedge$Quaker(Nixon)

$\forall$x Republican(x) : $\neg$Pacifist(x) / $\neg$Pacifist(x)

$\forall$x Quaker(x) : Pacifist(x) / Pacifist(x)

What should we conclude? 2 possible contradictory derivations, each blocking other out.

Sceptical queries and credulous queries:

- **Sceptical**: cannot derive anything about Nixon being a pacifist
- **Credulous**: pick any possible derivation

Circumscription: syntax

introduce "abnormal" predicates in rules (never asserted as facts)

$\forall x\ bird(x) \wedge \neg abnormal_1(x) \rightarrow canFly(x)$

bird(tweety),

bird(opus),

$\neg$canFly(opus)

Circumscription: semantics

what is the minimal set of “abnormal” facts that must be assumed to be true to make the KB consistent?

if we assume {$abnormal_1$(opus)}, then it works

convenient for large KBs where most objects are “normal”, but there are a few exceptions

the circumscription algorithm will figure out the minimal set that needs to be assumed abnormal

circumscription can be viewed as a form of “model preference”

of all possible models of some sentences, some are more plausible than others, i.e. the ones with fewer abnormal assumptions

sometimes even this is not enough to disambiguate the intended meaning...

perhaps we need to assign precedence among abnormal predicates...

Truth Maintenance Systems

in real-world applications, need to...

derive conclusions based on assumptions

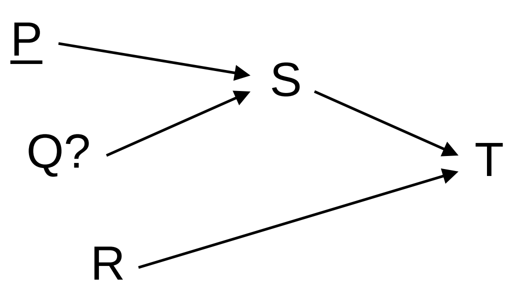
when conflicting information comes in (or facts change), need to change beliefs

if P changes from T to F, must identify and retract all consequences that depended on P

must keep track of network of *justifications*

TMS, JTMS: efficient algorithms for propagating changes in knowledge (minimal belief revision)

initially, I know P and R, and I assume Q is true, so I infer S and T



if later I come to find out that T is not true, then I reason backwards to identify that Q must not have been true, so I retract Q, S, and T (mark them as false)

Numeric probabilities:

Please first read a separate tutorial on probabilistic and fuzzy reasoning:

https://courses.cs.ttu.ee/w/images/b/b5/Uncertain_prob_fuzzy.ppt

and our presentation continues after that.

Combining numeric probabilities:

- Tourism recommenders as a case study
- Input and output
- Which probabilities we need?
- Simple layered semantics
- Cumulating evidence
- Rankings via meta-logical calculations

Recommender systems

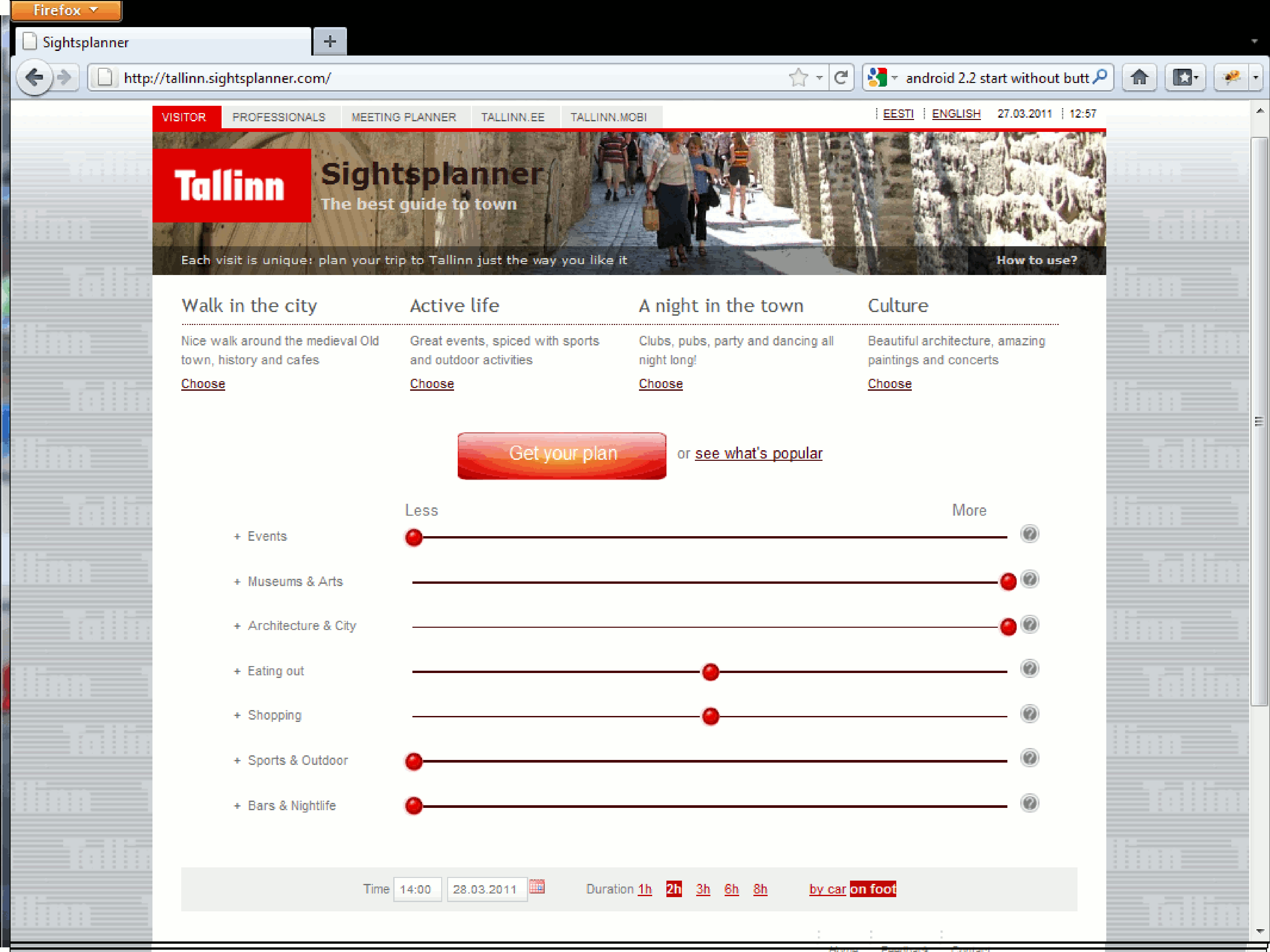
- Several historical „expert systems“ were recommender systems (medicine etc)
- Google is a popularity-focused recommender
- Social network systems are recommender systems: recommend news items and possible friends and topics
- The wealth of data available online makes it possible to create recommenders for any kinds of tasks and goals

Two main recommender types

- Collaborative filtering
- Rule-based, also called content-based

Our tourism recommender project

- http://www.sightsplanner.com
- http://www.sightsmap.com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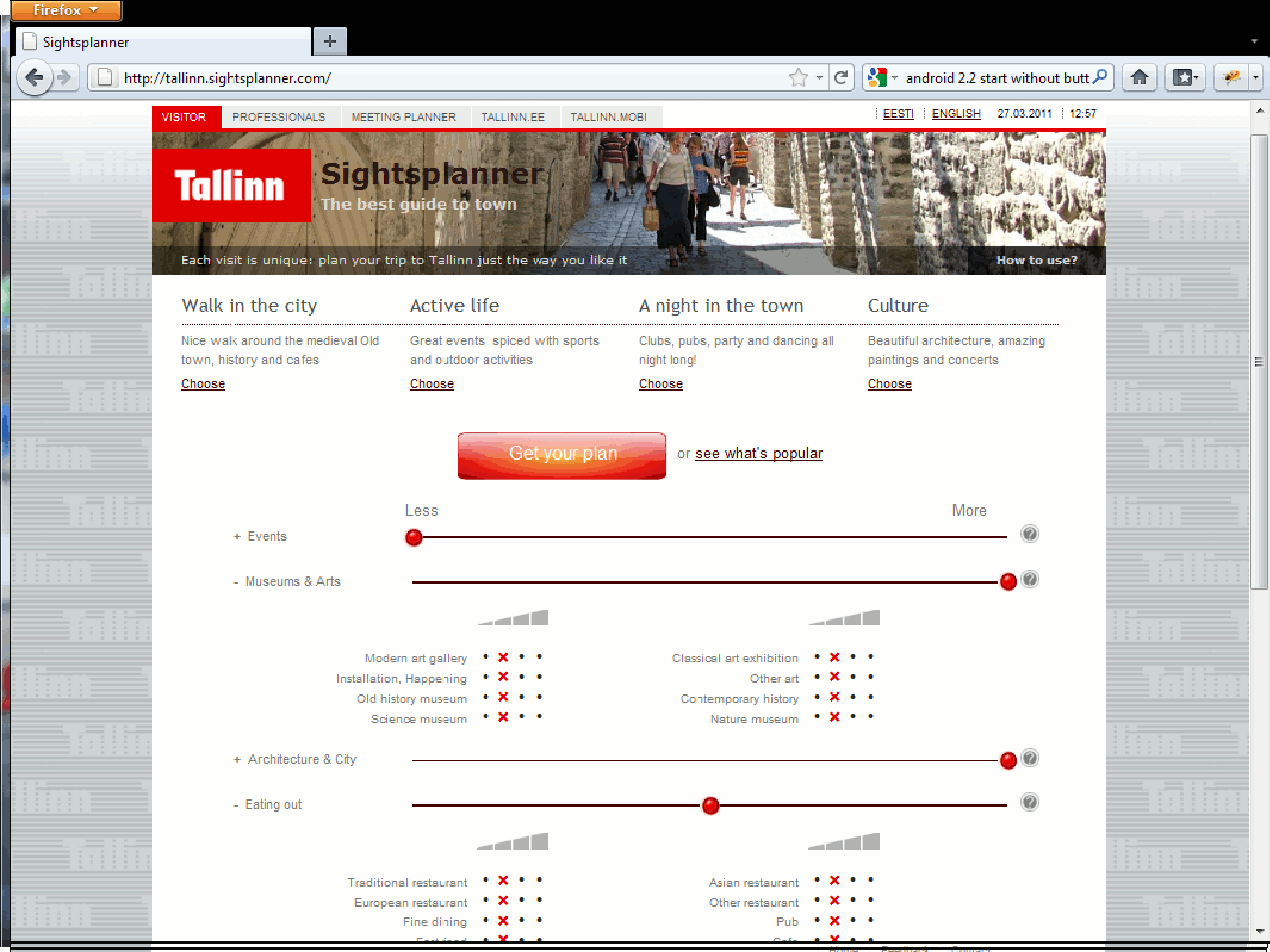
Tallinn
Sightsplanner
The best guide to town
Each visit is unique: plan your trip to Tallinn just the way you like it
How to use?
Walk in the city
Nice walk around the medieval Old town, history and cafes
Choose
Active life
Great events, spiced with sports and outdoor activities
Choose
A night in the town
Clubs, pubs, party and dancing all night long!
Choose
Culture
Beautiful architecture, amazing paintings and concerts
Choose
Get your plan
or see what's popular
Less
More
+ Events
+ Museums & Arts
+ Architecture & City
+ Eating out
+ Shopping
+ Sports & Outdoor
+ Bars & Nightlife
Time
14:00
28.03.2011
Duration
1h
2h
3h
6h
8h
by car
on foot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Choose
Get your plan
or see what's popular
Less
More
+ Events
- Museums & Arts
Modern art gallery
Installation, Happening
Old history museum
Science museum
Classical art exhibition
Other art
Contemporary history
Nature museum
+ Architecture & City
- Eating out
Traditional restaurant
European restaurant
Fine dining
Asian restaurant
Other restaurant
Pub

Firefox

Sightsplanner

http://tallinn.sightsplanner.com/maps/show

Time 14:00 29.03.2011 Duration 1h 2h **3h** 6h 8h by car **on foot**

1. Estonian History Museum - Great Guild Hall

Read more Locate on map

Museums and Arts | Architecture and City | Old history museum | Medieval architecture

In the course of time, the Great Guild Hall has played an important role in the life of the city. The permanent exhibition of the History Museum located in the building introduces Estonia's history...

Arrive 14:05
Stay
45
minutes
Remove from selection

Walk 5 minutes

2. Katariina käik (St Catherine's Passage)

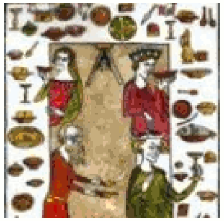
Read more Locate on map

Landmark | Architecture and City | Medieval architecture

Vene and Müürivahe Streets are connected by Katariina käik. You can see the remaining parts of St Catherine's Church in its northern end The southern part of the passage is lined by residenti...

Arrive 14:55
Stay
20
minutes
Remove from selection

Walk 5 minutes



3. Olde Hansa

Read more Locate on map

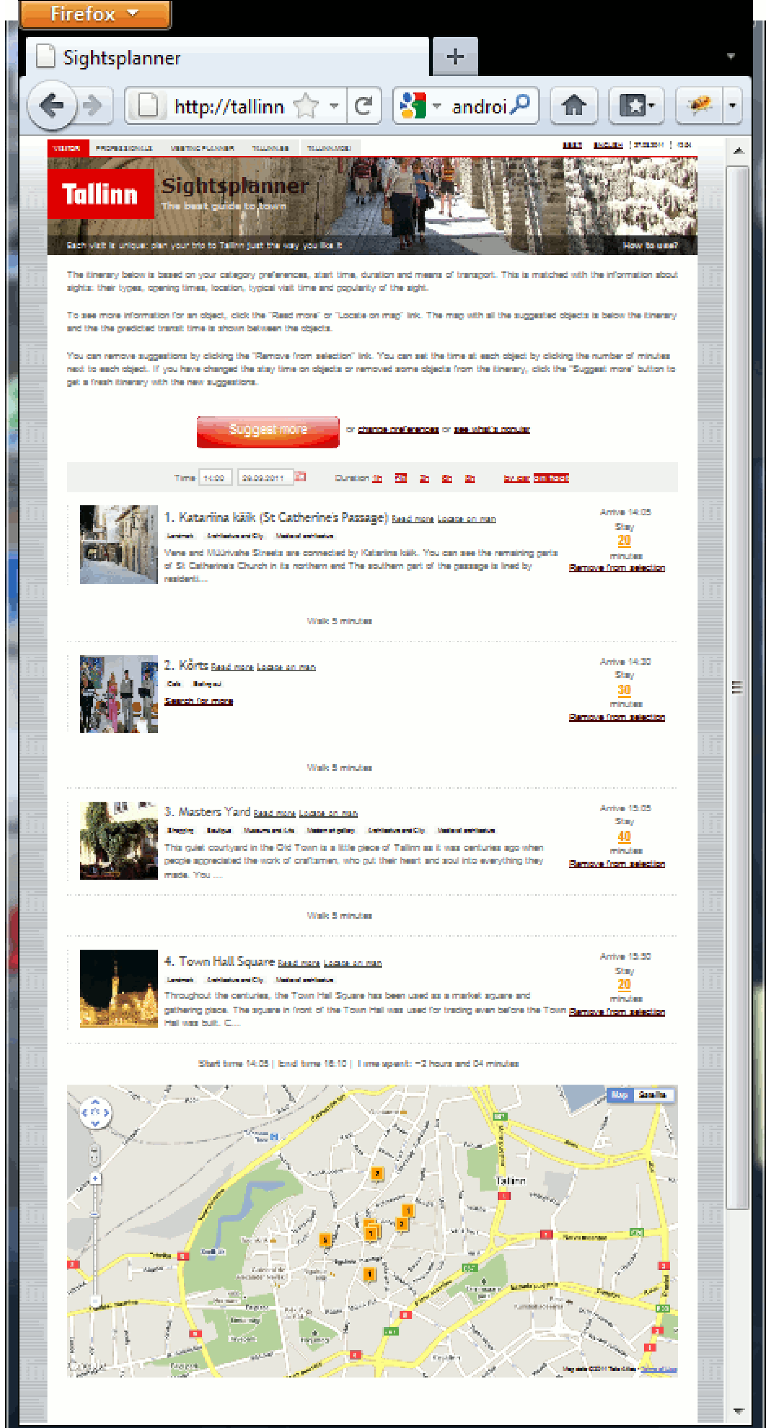
Eating out | Traditional restaurant

Olde Hansa Restaurant

See more

Arrive 15:20
Stay
60
minutes
Remove from selection

Walk 10 minutes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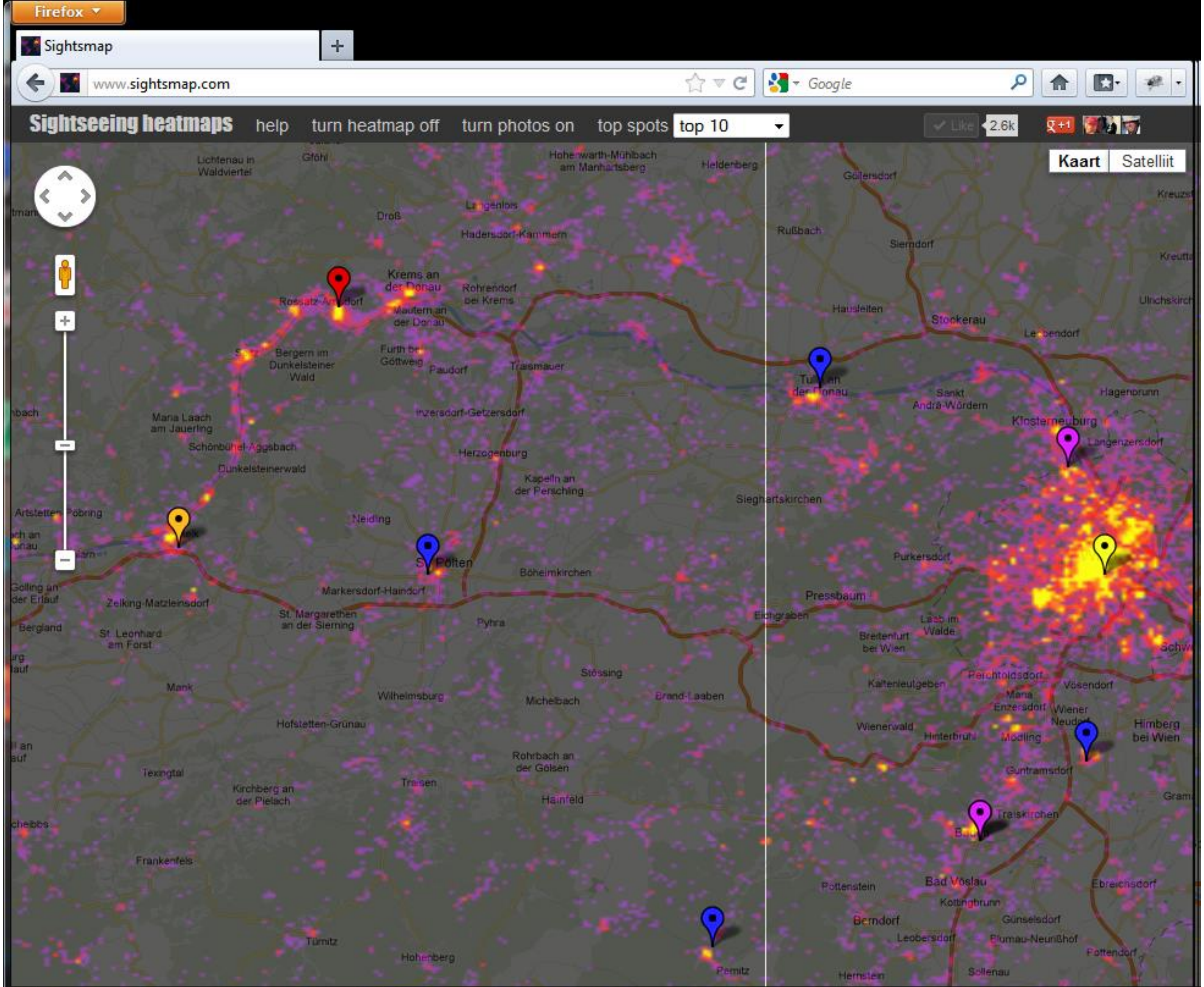
Tallinn
Sightsplanner
The best guide to town
Each visit is unique: plan your trip to Tallinn just the way you like it
How to use?
The itinerary below is based on your category preferences, start time, duration and means of transport. This is matched with the information about sights: their types, opening times, location, typical visit time and popularity of the sight.
To see more information for an object, click the "Read more" or "Locate on map" link. The map with all the suggested objects is below the itinerary and the the predicted transit time is shown between the objects.
You can remove suggestions by clicking the "Remove from selection" link. You can set the time at each object by clicking the number of minutes next to each object. If you have changed the stay time on objects or removed some objects from the itinerary, click the "Suggest more" button to get a fresh itinerary with the new suggestions.
Suggest more
Time
Duration
1. Katariina käik (St Catherine's Passage)
Arrive 14:05
Stay
20
minutes
Remove from selection
Walk 5 minutes
2. Kõrts
Search for more
Arrive 14:30
Stay
30
minutes
Remove from selection
Walk 5 minutes
3. Masters Yard
This quiet courtyard in the Old Town is a little piece of Tallinn as it was centuries ago when people appreciated the work of craftsmen, who put their heart and soul into everything they made. You ...
Stay
40
minutes
Remove from selection
Walk 5 minutes
4. Town Hall Square
Throughout the centuries, the Town Hall Square has been used as a market square and gathering place. The square in front of the Town Hall was used for trading even before the Town Hall was built. C...
Stay
20
minutes
Remove from selection
Tallinn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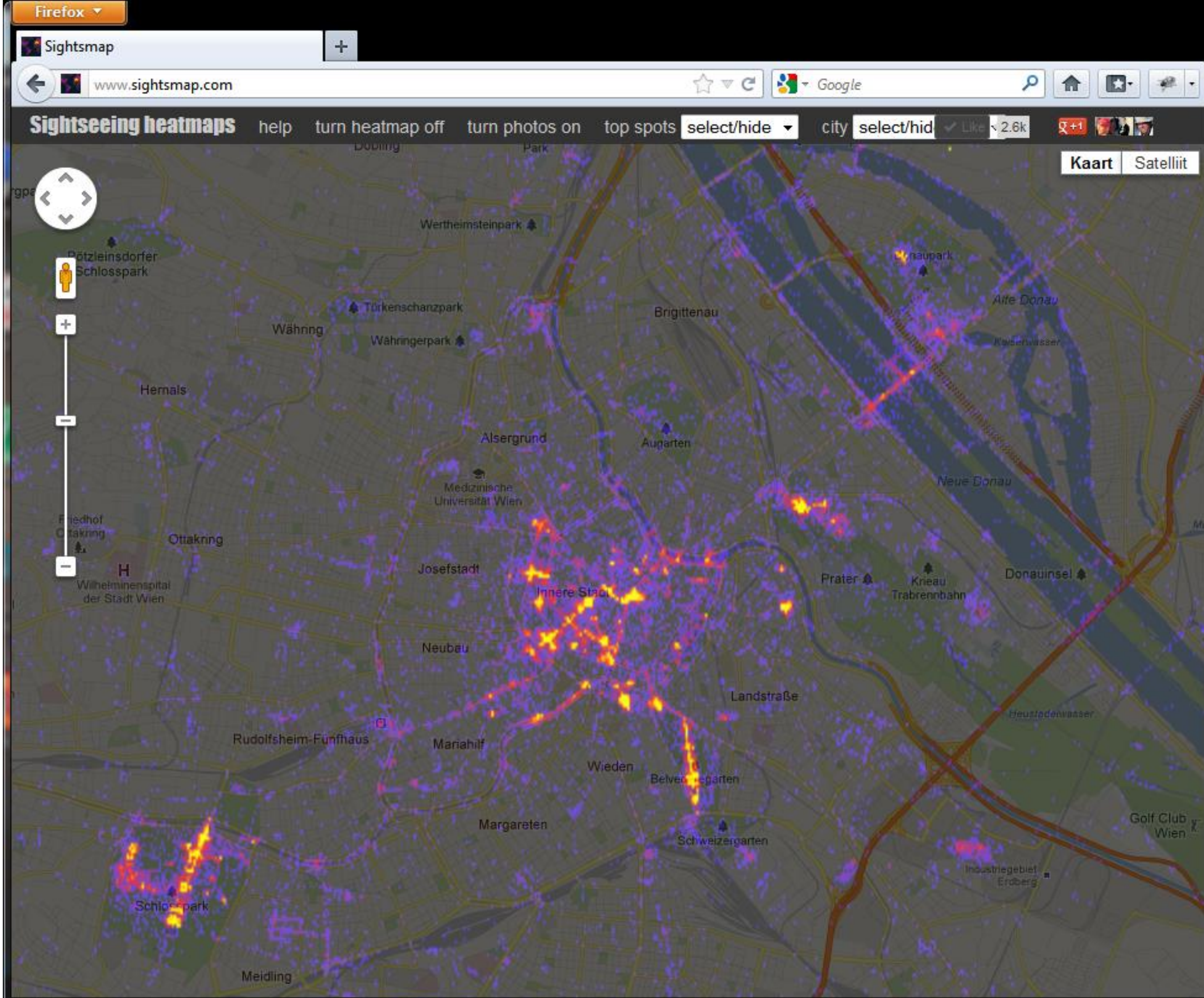
2.6k
Kaart
Satelliet
Lichtenau in Waldviertel
Gföhl
Hohenwarth-Mühlbach am Manhartsberg
Heldenberg
Göllersdorf
Droß
Hadersdorf-Kammern
Rußbach
Sierndorf
Krems an der Donau
Rohrendorf bei Krems
Mautern an der Donau
Hausleiten
Stockerau
Bergern im Dunkelsteiner Wald
Furth bei Göttweig
Paudorf
Traismauer
Sankt Andrä-Wördern
Hagenbrunn
Maria Laach am Jauerling
Schönbühel-Aggsbach
Dunkelsteinerwald
Herzogenburg
Kapelln an der Perschling
Sieghartskirchen
Klosterneuburg
Langenzersdorf
Neidling
Böheimkirchen
Purkersdorf
Pressbaum
Markersdorf-Haindorf
Zelking-Matzleinsdorf
St. Margarethen an der Sierning
Pyhra
Eichgraben
Breitenfurt bei Wien
Laab im Walde
Bergland
St. Leonhard am Forst
Mank
Stössing
Wilhelmsburg
Michelbach
Brand-Laaben
Kaltenleutgeben
Perchtoldsdorf
Vösendorf
Hofstetten-Grünau
Wienerwald
Hinterbrühl
Mödling
Himberg bei Wien
Texingtal
Kirchberg an der Pielach
Traisen
Rohrbach an der Gölsen
Hainfeld
Guntramsdorf
Traiskirchen
Frankenfels
Pottenstein
Bad Vöslau
Kottingbrunn
Ebreichsdorf
Berndorf
Günselsdorf
Leobersdorf
Blumau-Neurißhof
Türnitz
Hohenberg
Pernitz
Pottendorf
Sollenau
Hernstein



Firefox
Sightsmap
www.sightsmap.com
Google
Sightseeing heatmaps
help
turn heatmap off
turn photos on
top spots
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city
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Wertheimsteinpark
Pötzleinsdorfer Schlosspark
Türkenschanzpark
Brigittenau
Währing
Währingerpark
Alte Donau
Kaiserwasser
Hernals
Alsergrund
Augarten
Medizinische Universität Wien
Neue Donau
Friedhof Ottakring
Ottakring
Josefstadt
Innere Stadt
Prater
Krieau Trabrennbahn
Donauinsel
Wilhelminenspital der Stadt Wien
Neubau
Landstraße
Heustadelwasser
Rudolfsheim-Fünfhaus
Mariahilf
Wieden
Margareten
Schweizergarten
Golf Club Wien
Industriegebiet Erdberg
Meidling

Input 1

- User interests:

likes(john,nightlife,0.6)

likes(john,sports,0.8)

likes(john,music,0.7)

likes(john,heavymetal,0.9)

dislikes(john,classicalmusic,0.9)

Input 2

- Object properties:

type(omalley,bar,0.9)
activity(omalley,footballwatching,0.7)
popularity(omalley,1000)

type(crown,restaurant,1.0)
activity(crown,heavymetal,0.8)
popularity(crown,1500)
opentime(crown,12.00,0.9)

Input 3

- Knowledge about the world:

type(X,church,M) -> type(X,architecture,M*0.9)
type(X,bar,M) -> type(X,drinkingplace,M)
type(X,restaurant,M) -> type(X,drinkingplace,M*0.7)
activity(X,footballwatching,M) -> activity(X,sports,M)

type(X,fastfood,M) -> visitminutes(X,20,0.8*M)
type(X,bar,M) & M>0.75 -> openat12(X,0.85)

description(X,S) & contains_str(S ,“paintings“) & contains_str(S ,“gallery“) ->
  type(X,artcollection,0.8)

Output

- Recommendations: numerical ranks for all tourism objects:

rank(john,omalley,0.6)

rank(john,crown,0.5)

Reasoning tasks

- Object identities: are two objects A and B obtained from different sources actually equal?
- Object types from content: using title, abstract, source etc, calculate wheather the object is a city, a castle, a church, medieval, modern, a drama play, a classical music concert, a rock concert, ...
- Generalised object types: if we know that an object is a bar (with some confidence X), then it is also a nightlife spot (with some confidence Y)
- Additional properties like time of visit, opening times
- How well does an object match user preferences

Probabilities?

There is a large number of probability-oriented theories and several reasoning systems, yet no "mainstream" probabilistic rule-based derivation algorithms exist

Fuzzy logic, probabilistic logic, Bayes networks, ...

Probabilistic datalog, probabilistic prolog, ...

Mycin, Emycin, Cadiag-2, ...

Goal

Formulate a practical, correct and complete way to use probabilities in rules for the (tourism) recommender context, using object logic.

Metalogic:
0.9: type(X,church) -> type(X,architecture)
0.8: type(X,fastfood) -> visitminutes(X,20)

Object logic:
type(X,church,M) -> type(X,architecture,M*0.9)
type(X,fastfood,M) -> visitminutes(X,20,M*0.8)

Which kinds of probabilities?

Non-strict sets a la „blue“, „large“, ...

Fuzzy logic : p(A v B) = max(p(A),p(B))

0.95: type(X,church) -> type(X,architecture)

0.7: type(X,theatre) -> type(X,architecture)

Incomplete knowledge a la „not sure that“ ...

Probabilistic: p(A v B) = p(A)+p(B) – (p(A)*p(B))

0.8: type(X,bar) -> openat12(X)

Object logic:

type(X,church,M) -> type(X,architecture,M*0.9)

type(X,fastfood,M) -> visitminutes(X,20,M*0.9)

Object logic layers of interpretation

Pred(t): Pred(t) holds.

Pred(t,m): Pred(t) holds with a fuzzy measure at least m.

Pred(t,m,c): With confidence (probability) at least c, Pred(t) holds with at least a fuzzy measure m.

Pred(t,m,c,d): The fact "with confidence (probability) at least c, Pred(t) holds with at least a fuzzy measure m„ holds and depends on the set of clauses d.

Examples

bar(malloy,0.9,1): we are certain that malloy is bar with a fuzzy measure at least 0.9

bar(crown,0.9,0.8): we are 0.8 confident that crown is a bar with a fuzzy measure at least 0.9

Rule examples

bar(X,M,C) & M>L -> openat12(X,1,C*0.8):

when we have confidence C in that X is a bar with a measure M at least L, we are C*0.8 confident that it is open at 12 with a measure 1.

optionally
bar(X,M,C) -> openat12(X,1,M*C*0.8):

example of a sure rule:

bar(X,M,C) -> can_eat_at(X,M*0.5,C):

Fuzzy part is easy

Use your own preferred function f and limits for fuzzy derivation

Pred(X,M1) & Pred(X,M2) -> Pred(X, f(M1,M2))

Pred(X,M) & M>L -> Pred(X, f(M))

Standard derivation rules in resolution hold, nothing is added.

We can enhace subsumption, provided f is monotonic:

Pred(X,M1) subsumes Pred(Y,M2) iff Y=Xs and M1>=M2.

Probabilistic part requires tracking

Recall P(t,M,C,D): C is the probability and D is the set of facts on which the atom depends upon.

Always use rules of form

P(....,D1) & ... & P(...,Dn) & A1 & .... & An ->
P(....,union(D1,...,Dn))

where P atoms do contain probabilities and
A1 ... An do not contain probabilities

Multiplying probabilities

Generally the rules should have a form

P1(t1,M1,C1,D1) & ... & Pn(tn,Mn,Cn,Dn) ->
P(t,M,f(M1,...,M2),g(C1,...,Cn,D1,...,Dn),union(D1,...,Dn))

- In simple cases g(C1,...,Cn,D1,...,Dn) = C1*...*Cn
- However, if intersection(D1,....,Dn) is not empty, Ci-s corresponding to Di-s with multiple occurrences should be used only once

Cumulating evidence

Use evidence cumulating rule schema:

Pred(X,M1,C1,D1) & Pred(X,M2,C2,D2) &
Empty(Intersection(D1,D2))
->
Pred(X,min(M1,M2),(C1+C2)-(C1*C2),union(D1,D2))

Cumulating evidence

Example: independent facts

a) bar(X,M,C,D) & M>0.75 -> openat12(X,1,C*0.8,D)
b) intitle(X,"allnight",M,C,D) & M>0.75 -> openat12(X,1,C*0.9,D)
c) bar(malloy,1,1,{c}).
d) intitle(malloy,"allnight",1,1,{d}).

a,c: e) openat12(malloy,1,0.8,{c})
b,d: f) openat12(malloy,1,0.9,{d})

giving for our case (0.8+0.9=1.7, 0.8*0.9=0.72, 1.7-0.72=0.98)
openat12(malloy,1,0.98,{c,d})

Cumulating evidence

Example: dependent facts

f) activity(X,heavymetal,1,1,D) -> activity(X,music,1,1,D).

g) activity(X,Y,M1,C1,D1) & likes(U,Y,M2,C2,D2) ->
fits(U,X,1,M1*M2*C1*C2,union(D1,D2))

a) likes(john,music,1,0.6,{a})

b) likes(john,heavymetal,1,0.8,{b})

c) activity(crown,heavymetal,1,1,{c}).

c,f: h) activity(crown,music,1,1,{e}).

g,a,h(cf): i) fits(john,crown,1,0.6,{a,c})

g,b,c: j) fits(john,crown,1,0.8,{b,c})

Cumulating prohibited, since i and j share c

Ranking calculation in meta-logic

- Derive all open-at-time facts.
- Derive all independent addrank facts, using:

Popularity(X,P) -> addrank(X,pf(P))

Likes(X,Y,M1) & assoc(Z,Y,M2,C,D) ->
    addrank(X,Z,f(M1,M2,C),D)
Dislikes(X,Y,M1) & assoc(Z,Y,M2,C,D) ->
    addrank(X,Z,nf(M1,M2,C),D)

- Sum all maximal pos/neg addrank numbers for objects.
- Filter out objects which are open at time.
- Order by rank.

Summary 1

Represent facts as P(t,M,C,D) where:

M- fuzzy measure of P(t) holding

C – confidence as probability of at least P(t,M) holding

D – set of facts on which P(t,M,C) depends

Represent rules as

P1(t1,M1,C1,D1) & ... & Pn(tn,Mn,Cn,Dn) &

M1>L1 & ... & Mn>Ln & A1 .... & Am

->

P(t,M,f(M1,...,M2),g(C1,...,Cn,D1,...,Dn),union(D1,...,Dn))

Summary 2

Add evidence cumulating rule

Pred(X,M1,C1,D1) & Pred(X,M2,C2,D2) &
Empty(Intersection(D1,D2))
->
Pred(X,min(M1,M2),(C1+C2)-(C1*C2),union(D1,D2))

Add extended subsumption

Pred(X,M1,C1,D1) subsumes
Pred(Y,M2,C2,D2)
iff Y=Xs & M1>=M2 & C1>=C2 &
D1 is a subset of D2