Exercise 1

```
{N>0 /\ N=n}                                   % ≡ Pre
    Z:=1
{Pre /\ Z=1}                                   % annotation
    WHILE N > 0 DO
                {N ≥ 0 /\ Z * M^N = M^n}       % ≡ Inv
          BEGIN
            Z := Z * M;
            N := N - 1;
          END;
{Z= M^n}                                       % ≡ Post
```

Exercise 2

Prove partial correctness of the guarded command program specification:

$$\{x \geq 0\}\ \ y := 1; \star[y \times y \leq x \rightarrow y := y+1]; y := y-1\ \ \{y^2 \leq x < (y+1)^2\}$$

Hint: try with invariant $(y - 1)^2 \leq x$

Exercise 3

Given an annotated program $S_1 \| S_2$ verify if it is interference free and prove the partial correctness of S_1 and S_2 separately.

$P_1 \equiv \{x \leq 4 \wedge y = 2\}$

S_1: $\langle x \geq 2 \rightarrow y := y - 2 \rangle$

$Q_1 \equiv \{y \leq x \wedge x \geq 0\}$

$\|$

$P_2 \equiv \{x \geq 0 \wedge y \geq 0\}$

S_2: $\langle x = 4 \wedge y = 1 \rightarrow z := x - 3 \rangle$

$Q_2 \equiv \{y + 2 \leq x\}$

Exercise 4

Specify the cooperation tests for channels C and D

$P \equiv \{x = 6 \wedge u = 0 \wedge y - x = 6\}$

$P_1 \equiv \{x >= 5 \wedge y > 7\}$

$S_1 : \langle C!\ x + 3 \rangle;\ \{x > 0 \wedge y \geq 7\} \langle D?\ y \rangle;\ \{y > 10 \wedge x > 0\}$

$Q_1 \equiv \{y > 9 \wedge x > 0\}$

$\|$

$P_2 \equiv \{u = 0\}$

$S_2 : \langle C?\ u \rangle;\ \{u = 8\} \langle D!\ u+5 \rangle$

□

$S_2 : \langle C?\ u \rangle;\ \langle u := u - 1 \rangle \{u = 7\} \langle D!\ u+5 \rangle$

$Q_2 \equiv \{u < 9\}$

$Q \equiv \{x > 0 \wedge u < 10 \wedge y > 0\}$

Exercise 5

Let R and T be nonempty sets of natural numbers. Consider the following partitioning algorithm $S_1 \| S_2$, where

$$S_1 \equiv max := max(R); c?mn; d!max;$$
$$\star[max > mn \rightarrow R := (R \setminus \{max\}) \cup \{mn\}; max := max(R); c?mn; d!max]$$

$$S_2 \equiv min := min(T); c!min; d?mx;$$
$$\star[mx > min \rightarrow T := (T \setminus \{min\}) \cup \{mx\}; min := min(T); c!min; d?mx]$$

Prove, by means of the method of Levin & Gries,

$\{R = R_0 \neq \emptyset \wedge T = T_0 \neq \emptyset \wedge R \cap T = \emptyset\}\ \ S_1 \| S_2$

$\{|R| = |R_0| \wedge |T| = |T_0| \wedge R \cup T = R_0 \cup T_0 \wedge max(R) < min(T)\}$

where, for a set A, $|A|$ denotes the number of elements of A, and R_0 and T_0 are logical variables denoting a finite set of natural numbers.