Exercises

Exercise 1. Show that:

1. Set $\mathbb{Z}$ is countably infinite
2. The set of even integers is countably infinite
3. The set of integers in the form 10^x, where $x \in \mathbb{Z}$ is countably infinite
4. There are as many odd integers as there are even integers
5. There is as many even numbers as there are integers

Exercise 2. Determine which of the following functions are injective and which are surjective.

(a) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$.

(b) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n^2 + 3$.

(c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin x$.

(d) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = x^2$.

(e) $f : \mathbb{Z} \to \mathbb{Q}$ defined by $f(n) = n/1$.

(f) $f : \mathbb{Q} \to \mathbb{Z}$ defined by $f(p/q) = p$, where p/q is a rational number expressed in its lowest terms with a positive denominator.

(g) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x$.

Exercise 3. Is relation $f \subseteq \mathbb{Q} \times \mathbb{Z}$ given by $f\left(\frac{p}{q}\right) = p$ a function?

Exercise 4. Is the relation $f \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $f(x) = 2x$ a function?

Exercise 5. Is the relation $f \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $f(x) = \frac{x}{2}$ a function?

Exercise 6. Which of the following relations $f \subseteq \mathbb{Q} \times \mathbb{Q}$ define a function? If f is not a function, supply a reason for it.

$$(a) \quad f\left(\frac{p}{q}\right) = \frac{p+1}{p-2} \qquad (b) \quad f\left(\frac{p}{q}\right) = \frac{3p}{2q}$$

$$(c) \quad f\left(\frac{p}{q}\right) = \frac{p+q}{q^2} \qquad (d) \quad f\left(\frac{p}{q}\right) = \frac{3p^2}{7q^2} - \frac{p}{q}$$

Exercise 7. Given a permutation

$$\pi = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

on a set $S = \{1, 2, 3\}$, define an inverse permutation π^{-1}.

Exercise 8. Let $f(x) = x^2$ and $g(x) = 2x + 5$. Define compositions $(f \circ g)(x)$ and $(g \circ f)(x)$. Are they the same?

Exercise 9. Let $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$. Define compositions $(f \circ g)(x)$ and $(g \circ f)(x)$. Are they the same?

Exercise 10. Let $h : S \to T$ be a bijection, and let h^{-1} be its inverse. What are the mappings $h \circ h^{-1}$ and $h^{-1} \circ h$?