

# OpenJML and SMT solvers

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# Precursors to OpenJML

- JML was first used in an early *extended static checker* (ESC/Java) and was implemented in a set of tools called JML2.
- The second generation of ESC/Java, ESC/Java2, was made current with Java 1.4 and with the definition of JML.
- JML2 tools were based on hand-crafted compilers and the maintenance and update effort was overwhelming as Java evolved.
- A new approach was needed, one that built on an existing compiler to leverage further developments in that compiler but *allowed easy integration* with a Java IDE environment, and was *readily maintainable and extensible*.

# OpenJML - Introduction

- OpenJML is an implementation of JML tools built by extending the OpenJDK Java tool set.
- OpenJDK has a readily extensible architecture, although it is quite amenable to extension since it has a complex compilation process with many components.
- The result is a suite of JML tools for **Java 8** that provides *static analysis*, *specification documentation*, and *runtime checking*, an API that is used for other tools, uses Eclipse as an IDE, and can be extended for further research.
- The main drawback is that in an Eclipse-integrated system, the Eclipse compiler is used (as is) for Java compilation and the OpenJML/OpenJDK compiler is used as a back-end tool for handling JML and verification tasks.

# OpenJML command-line tool

- Ability to parse and type-check current JML
- Ability to perform static verification checks using back-end SMT solvers
- Ability to explore counterexamples (models) provided by the solver
- Partial implementation of JML-aware documentation generation
- Proof of concept implementation of runtime assertion checking
- JMLUnitNG has used OpenJML to create a test generation tool, using OpenJML's API to access the parsed specifications

# Eclipse Java development environment OpenJML plug-in

- Ability to parse and type-check JML showing any errors or warnings as Eclipse problems, but with a custom icon and problem type
- Ability to check JML specifications against the Java code
  - Verification conditions are produced from the internal **ASTs** (Abstract Syntax Trees) and submitted to a back-end **Satisfiability Modulo Theories (SMT)** solver, and any proof failures are shown as Eclipse problems.
- Ability to use files with runtime checks along with Eclipse-compiled files
- Ability to explore specifications and counterexamples *within* the GUI
- Functionality integrated as Eclipse menus, commands, and editor windows

# Exploring Counterexamples from Static Checking

- The Eclipse GUI enables exploring counterexamples produced by failed static checking much more effectively than previous JML tools.
- The Eclipse GUI for OpenJML interprets the counterexample information and relates it directly to the program as seen in the Eclipse editor windows.
- Previously, other tools created verification conditions, shipped them to a back-end solver, which produced counterexample information that was essentially a dump of the prover state and was notoriously difficult to debug.

# OpenJML back-end SMT solvers – What is SMT?

- SMT solvers are useful for verification, proving the correctness of programs, software testing based on symbolic execution, and for program synthesis.
- Computer-aided verification of computer programs often uses SMT solvers.
- In computer science and mathematical logic, the **Satisfiability Modulo Theories (SMT)** problem is a decision problem for logical formulas *with respect to combinations of background theories* expressed in classical *first-order logic with equality*.
  - Examples of theories: Real numbers, integers, theories of data structures like lists, arrays, bit-vectors, ...

# SMT Instances

- An SMT instance is a *formula in first-order logic*, where some function and predicate symbols have additional interpretations, and SMT is the problem of determining whether such a formula is satisfiable.
- An SMT instance is a *generalization of a Boolean SAT instance* in which various sets of variables are replaced by predicates from corresponding underlying theories.
  - **Boolean SAT problem** is the problem of determining if there exists an interpretation that satisfies a given Boolean formula, i.e., it asks whether the variables of a given Boolean formula can be consistently replaced by the values TRUE or FALSE in such a way that the formula evaluates to TRUE.
    - E.g., " $a$  AND NOT  $b$ " is satisfiable and " $a$  AND NOT  $a$ " is unsatisfiable.
- SMT formulas provide a much richer modeling language than is possible with Boolean SAT formulas.

# Verification and Testing with SMT Solvers

- For verification of programs a common technique is to translate *pre-conditions*, *post-conditions*, *loop conditions*, and *assertions* into SMT formulas in order to determine if all properties can hold.
- Another important application of SMT solvers is *symbolic execution* for *analysis* and *testing* of programs.

# OpenJML back-end SMT solvers

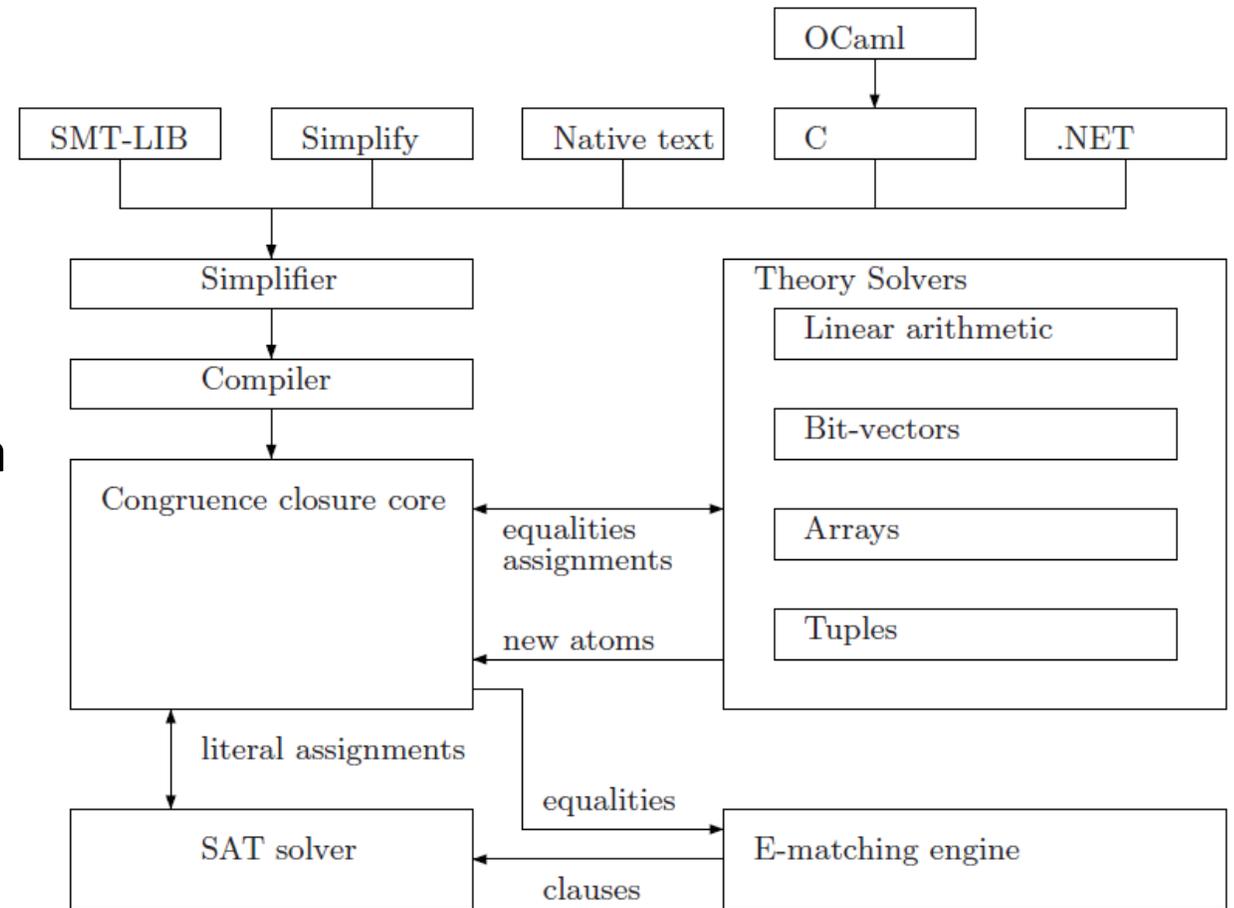
- OpenJML translates JML specifications into SMT-LIB format and passes the proof problems implied by the Java+JML program to back-end SMT solvers.
- OpenJML can use any SMT-LIBv2-compliant solver.
  - Z3, CVC4, Yices, ...
  - *Simplify* was the theorem prover of the *Extended Static Checkers* ESC/Java and still supported by OpenJML.
- Success in checking the consistency of the specifications and the code will depend on:
  - (a) the capability of the back-end SMT solver,
  - (b) the particular encoding of code + specifications into SMT by OpenJML, and
  - (c) the complexity and style in which the code and specifications are written.

# OpenJML Z3 back-end SMT solver

- Supported theories: *empty theory*, *linear arithmetic*, *nonlinear arithmetic*, *bit-vectors*, *arrays*, *datatypes*, *quantifiers*, *strings*
- Advanced algorithms for quantifier instantiation and theory combination.
- Z3 integrates a DPLL-based SAT solver, a core theory solver for equalities and uninterpreted functions, *satellite solvers* and an engine for quantifiers.

To get started:

<https://rise4fun.com/z3/tutorial/guide>



# OpenJML CVC4 back-end SMT solver

- CVC4 works with a version of first-order logic with *polymorphic* types.
  - <http://cvc4.cs.stanford.edu/web/>
- Several built-in base theories: rational and integer linear arithmetic, arrays, tuples, records, inductive data types, bit-vectors, strings, and equality over uninterpreted function symbols (“empty theory”).
- Support for quantifiers through heuristic instantiation.
- CVC4 is fundamentally similar to other modern SMT solvers like Z3: it is a DPLL solver, with a SAT solver at its core and a delegation path to different decision procedure implementations, each in charge of solving formulas in some background theory.

# OpenJML and Simplify theorem prover

- *Simplify* is a theorem prover for program checking developed at HP Labs.
- Simplify is the proof engine of the *Extended Static Checkers* ESC/Java.
- The goal of ESC is to prove, at compile-time, the absence of certain run-time errors, such as out-of-bounds array accesses and unhandled exceptions.
- The ESC approach first processes source code with a verification condition generator, which produces first-order formulas asserting the *absence* of the targeted errors, and then submits those verification conditions to the theorem prover.

# OpenJML and Simplify theorem prover (cont.)

- Input to *Simplify* is an arbitrary first-order formula, including quantifiers.
- *Simplify* handles propositional connectives by *backtracking search* and includes complete decision procedures for the supported theories (untyped first-order logic with function symbols and equality, arithmetic, maps, partial orders, ...).
- To test whether a formula is satisfiable, *Simplify* performs a backtracking search, guided by the propositional structure of the formula, attempting to find a satisfying assignment of truth values to atomic formulas that makes the formula *true*.

# OpenJML Yices 2 back-end SMT solver

- Yices 2<sup>1</sup> is an SMT solver that decides the satisfiability of formulas containing uninterpreted function symbols with equality, real and integer arithmetic, bit-vectors, scalar types, and tuples.
- Both linear and nonlinear arithmetic is supported.
- Yices 2 includes a congruence-closure algorithm inspired by Simplify's E-graph and used an approach for theory combination based on the Nelson-Oppen method (also used in Simplify and other SMT solvers like Z3) complemented with lazy generation of interface equalities.

<sup>1</sup> <http://yices.csl.sri.com/>

# OpenJML and Testing

- **JMLUnitNG**<sup>1</sup> is an automated unit test generation tool for JML-annotated Java code, including code using Java 1.5+ features such as generics, enumerated types, and enhanced for loops.
- JML assertions are used as *test oracles*.
- Tests can be generated for OpenJML RAC.
- Testing a class (or set of classes) with JMLUnitNG involves:
  1. Generating the test classes
  2. Compile the classes under test with OpenJML
  3. Compile the generated (test) classes with a regular Java compiler
  4. Run the tests.

<sup>1</sup> <http://insttech.secretninjaformalmethods.org/software/jmlunitng/>

The following slides are based on material presented by Leonardo de Moura and Nikolaj Bjørner in various presentations on Z3, found on:

<http://leodemoura.github.io/slides.html>

# Basics - Language of logic

- Functions , Variables, Predicates
  - $f, g, \quad x, y, z, \quad P, Q, =$
- Atomic formulas, Literals
  - $P(x, f(y)), \neg Q(y, z)$
- Quantifier free formulas
  - $P(f(a), b) \wedge c = g(d)$
- Formulas, sentences
  - $\forall x . \forall y . [ P(x, f(x)) \vee g(y, x) = h(y) ]$

# Language: Signatures

- A *signature*  $\Sigma$  is a finite set of:

- Function symbols:

$$\Sigma_f = \{ f, g, \dots \}$$

- Predicate symbols:

$$\Sigma_p = \{ P, Q, =, \text{true}, \text{false}, \dots \}$$

- And an *arity* function:

$$\Sigma \rightarrow \mathbb{N}$$

- Function symbols with arity 0 are *constants*
- A countable set  $V$  of *variables*
  - disjoint from  $\Sigma$

# Language: Quantifier free formulas

- The set  $\text{QFF}(\Sigma, V)$  of *quantifier free formulas* is the smallest set such that:

$\varphi \in \text{QFF}$	$::= a \in \text{Atoms}$	<i>atoms</i>
	$  \neg \varphi$	<i>negations</i>
	$  \varphi \leftrightarrow \varphi'$	<i>bi-implications</i>
	$  \varphi \wedge \varphi'$	<i>conjunction</i>
	$  \varphi \vee \varphi'$	<i>disjunction</i>
	$  \varphi \rightarrow \varphi'$	<i>implication</i>

# Language: Formulas

- The set of *first-order formulas* are obtained by adding the formation rules:

$\varphi ::= \dots$

|  $\forall x . \varphi$                     *universal quant.*

|  $\exists x . \varphi$                     *existential quant.*

- *Free* (occurrences) of *variables* in a formula are those not bound by a quantifier.
- A *sentence* is a first-order formula with no free variables.

# Theories

- A (first-order) *theory*  $T$  (over signature  $\Sigma$ ) is a set of (deductively closed) sentences (over  $\Sigma$  and  $V$ )
- Let  $DC(\Gamma)$  be the *deductive closure* of a set of sentences  $\Gamma$ .
  - For every theory  $T$ ,  $DC(T) = T$
- A theory  $T$  is *consistent* if  $false \notin T$
- We can view a (first-order) theory  $T$  as the class of all *models* of  $T$  (due to completeness of first-order logic).

# Models (Semantics)

- A model  $M$  is defined as:
  - Domain  $S$ ; set of elements.
  - Interpretation,  $f^M : S^n \rightarrow S$  for each  $f \in \Sigma_F$  with  $\text{arity}(f) = n$
  - Interpretation  $P^M \subseteq S^n$  for each  $P \in \Sigma_P$  with  $\text{arity}(P) = n$
  - Assignment  $x^M \in S$  for every variable  $x \in V$
- A *formula*  $\varphi$  is true in a model  $M$  if it evaluates to true under the given interpretations over the domain  $S$ .
- $M$  is a *model for the theory*  $T$  if all sentences of  $T$  are true in  $M$ .

# T-Satisfiability

- A formula  $\varphi(x)$  is T-satisfiable in a theory  $T$  if there is a model of  $DC(T \cup \exists x \varphi(x))$ . That is, there is a model  $M$  for  $T$  in which  $\varphi(x)$  evaluates to true.
- Notation:

$$M \models_T \varphi(x)$$

# T-Validity

- A formula  $\varphi(x)$  is *T-valid* in a theory  $T$  if  $\forall x \varphi(x) \in T$ .  
That is,  $\varphi(x)$  evaluates to *true* in every model  $M$  of  $T$ .
- *T-validity*:

$$\models_T \varphi(x)$$

# Checking Validity – the morale

- Theory solvers must minimally be able to:
  - check *unsatisfiability* of conjunctions of literals.

# Clauses – CNF conversion

Generally SMT solvers work with formulas in *Conjunctive Normal Form* (CNF).

$\varphi : x = 5 \Leftrightarrow (y < 3 \vee z = x)$  *is not in CNF.*

# Clauses – CNF conversion

$$\varphi : x = 5 \Leftrightarrow (y < 3 \vee z = x)$$



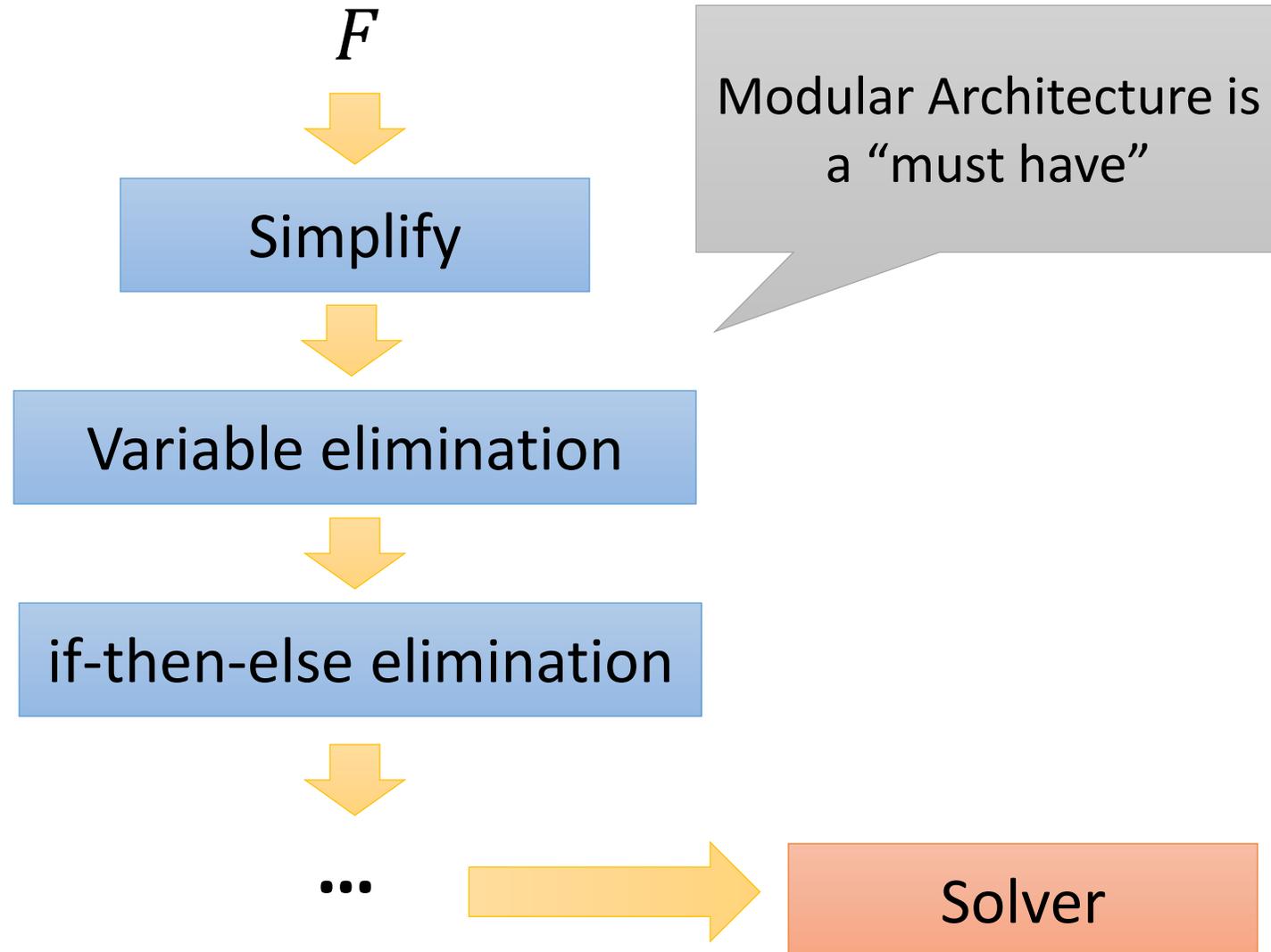
Equi-satisfiable CNF formula

$$\begin{aligned} \varphi' : & (\neg p \vee x = 5) \wedge (p \vee \neg x = 5) \wedge \\ & (\neg p \vee y < 3 \vee z = x) \wedge \\ & (p \vee \neg y < 3) \wedge (p \vee \neg z = x) \end{aligned}$$

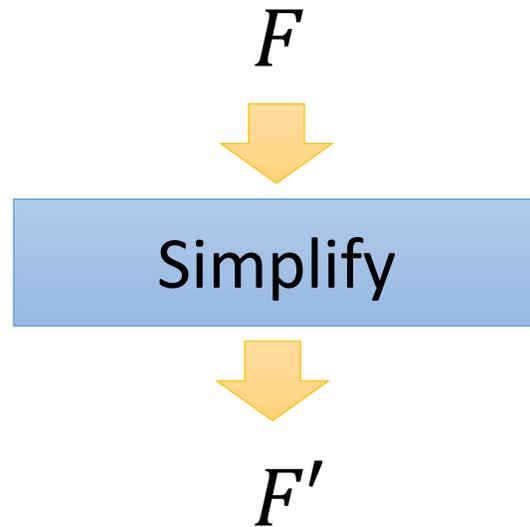
# Clauses - CNF

- Main properties of basic CNF:
  - Result  $F$  is a set of *clauses*.
  - $\varphi$  is  $T$ -satisfiable iff  $\text{CNF}(\varphi)$  is.
  - $\text{size}(\text{CNF}(\varphi)) \leq 4(\text{size}(\varphi))$
  - $\varphi \Leftrightarrow \exists p_{\text{aux}} \text{CNF}(\varphi)$

# Preprocessing of formulas for SMT solver



# Equivalence Preserving Simplifications



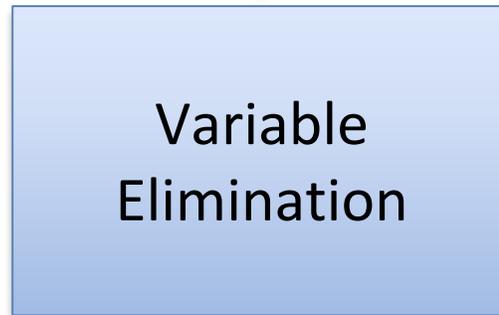
Examples:

$$x + y + 1 - x - 2 \mapsto y - 1$$

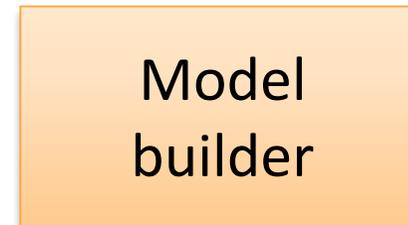
$$p \wedge \text{true} \wedge p \mapsto p$$

# Example

$[ a = b + 1, (a < 0 \vee a > 0), b > 3 ]$

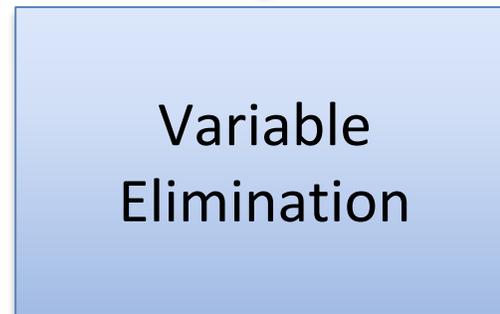


$[ (b + 1 < 0 \vee b + 1 > 0), b > 3 ]$



# Example

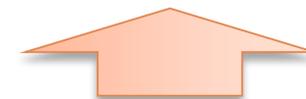
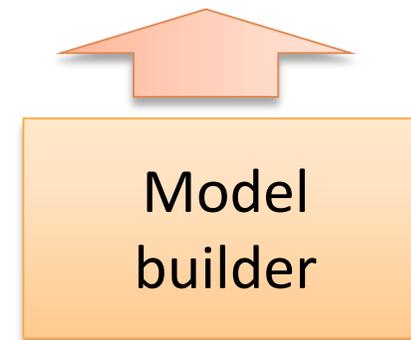
$$[ a = b + 1, (a < 0 \vee a > 0), b > 3 ]$$



$$M, M(a) = M(b) + 1$$



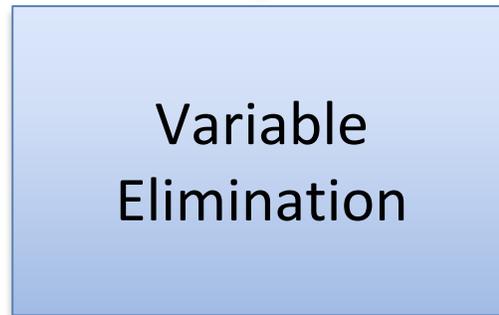
$$[ (b + 1 < 0 \vee b + 1 > 0), b > 3 ]$$



$M$

# Example

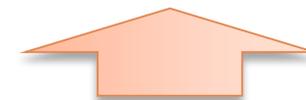
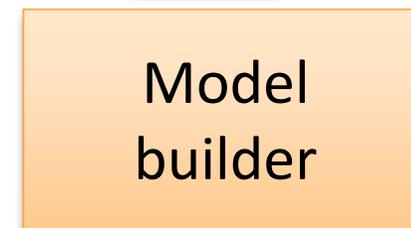
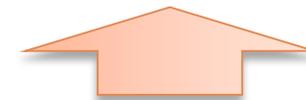
$$[ a = b + 1, (a < 0 \vee a > 0), b > 3 ]$$



$$[ (b + 1 < 0 \vee b + 1 > 0), b > 3 ]$$

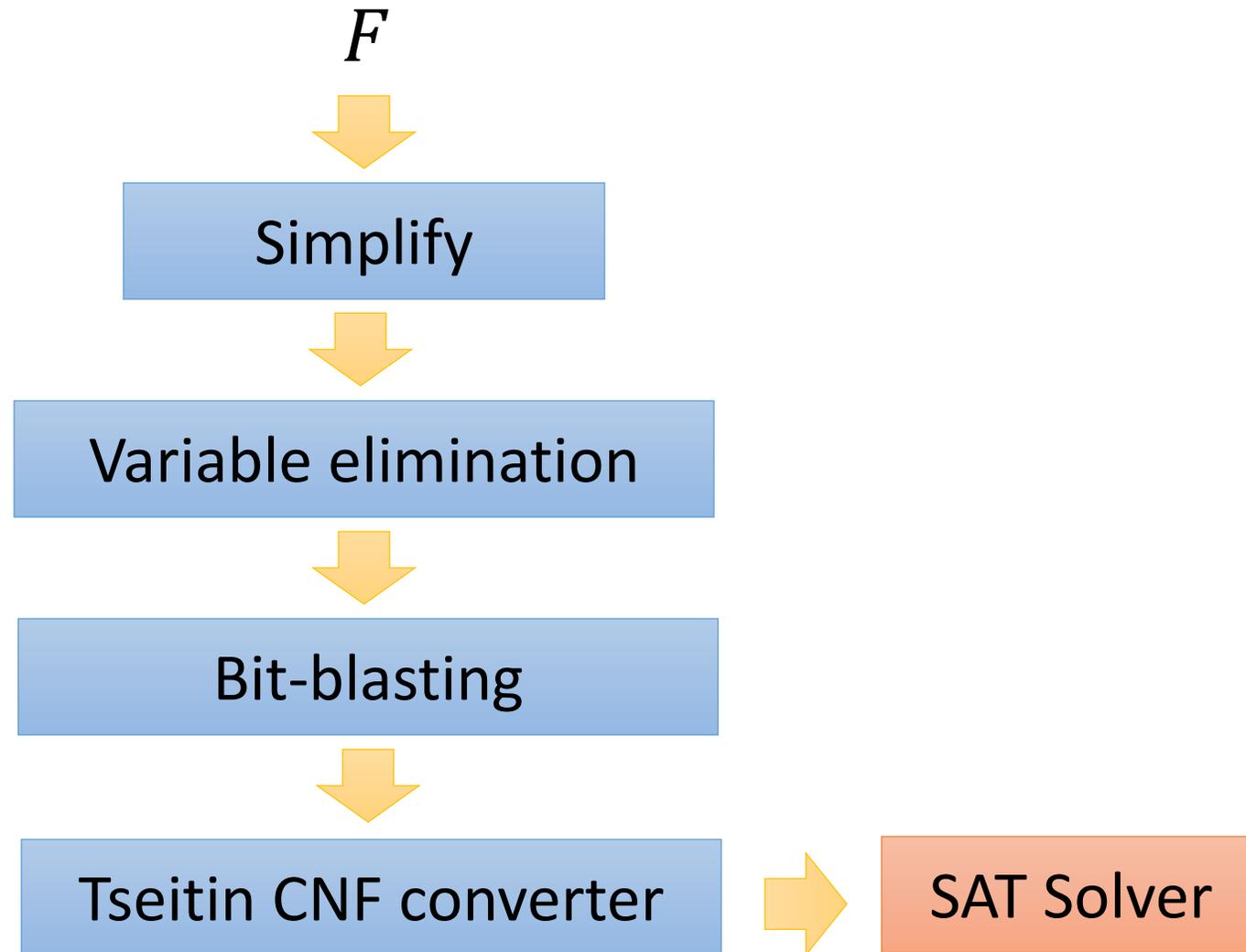


$$b \rightarrow 5, a \rightarrow 6$$



$$b \rightarrow 5$$

# Simple **QF\_BV** (bit-vector) solver



# Satisfiability Modulo Theories (SMT)

**Is formula  $F$  satisfiable  
modulo theory  $T$  ?**

SMT solvers have  
specialized algorithms for  $T$

# Z3

# Solver

DPLL

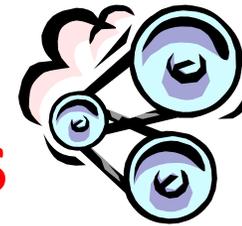
Simplex

Rewriting

Superposition



Z3 is a collection of  
**Symbolic Reasoning Engines**



Congruence  
Closure

Groebner  
Basis

$\forall\exists$   
elimination

Euclidean  
Solver

# Satisfiability Modulo Theories (SMT)

$$b + 2 = c \text{ and } f(\text{read}(\text{write}(a,b,3), c-2) \neq f(c-b+1)$$

# Satisfiability Modulo Theories (SMT)

$b + 2 = c$  and  $f(\text{read}(\text{write}(a, b, 3), c-2) \neq f(c-b+1)$

Arithmetic

# Satisfiability Modulo Theories (SMT)

$b + 2 = c$  and  $f(\text{read}(\text{write}(a,b,3), c-2) \neq f(c-b+1)$

Array Theory

# Satisfiability Modulo Theories (SMT)

$$b + 2 = c \text{ and } f(\text{read}(\text{write}(a, b, 3), c-2)) \neq f(c-b+1)$$

Uninterpreted  
Functions

# Approaches to linear arithmetic

- Fourier-Motzkin:

- Quantifier elimination procedure

$$\exists x (t \leq ax \wedge t' \leq bx \wedge cx \leq t'') \Leftrightarrow ct \leq at' \wedge ct' \leq bt''$$

- Polynomial for difference logic.
- Generally: exponential space, doubly exponential time.

- Simplex:

- Worst-case exponential, but
- Time-tried practical efficiency.
- Linear space

# Combining Theory Solvers: Nelson-Oppen procedure

**Initial state:**  $L$  is set of literals over  $\Sigma_1 \cup \Sigma_2$

**Purify:** Preserving satisfiability,  
convert  $L$  into  $L' = L_1 \cup L_2$  such that  
 $L_1 \in T(\Sigma_1, V)$ ,  $L_2 \in T(\Sigma_2, V)$   
So  $L_1 \cap L_2 = V_{\text{shared}} \subseteq V$

**Interaction:**

Guess a partition of  $V_{\text{shared}}$

Express the partition as a conjunction of equalities.

Example,  $\{x_1\}$ ,  $\{x_2, x_3\}$ ,  $\{x_4\}$  is represented as:

$$\psi: x_1 \neq x_2 \wedge x_1 \neq x_4 \wedge x_2 \neq x_4 \wedge x_2 = x_3$$

**Component Procedures:**

Use solver 1 to check satisfiability of  $L_1 \wedge \psi$

Use solver 2 to check satisfiability of  $L_2 \wedge \psi$

# Example Theory in Z3: Arrays

- Functions:  $\Sigma_f = \{ read, write \}$
- Predicates:  $\Sigma_p = \{ = \}$
- Convention  $a[i]$  means:  $read(a, i)$
  
- Non-extensional arrays  $T_A$ :
  - $\forall a, i, v. write(a, i, v)[i] = v$
  - $\forall a, i, j, v. i \neq j \Rightarrow write(a, i, v)[j] = a[j]$
  
- Extensional arrays:  $T_{EA} = T_A +$ 
  - $\forall a, b. ((\forall i. a[i] = b[i]) \Rightarrow a = b)$

# Suggested reading for JML and contracts

- Paper: "Design by Contract with JML".
- JML reference manual (updated occasionally)
- Book: Deductive Software Verification – The KeY Book
  - Chapters 3, 7 and 8 especially. Incrementally introducing more advanced concepts for JML.
- Paper: "Desugaring JML Method Specifications" for additional help for understanding of JML.
- Relevant papers on chosen tool (SMT Solver or other) by each group.