1. Write standard prime decomposition of 90.
2. Write the 6-adic expansion of 11.
3. How many elements of $\mathbb{Z}_{99}$ are invertible under multiplication?
4. Express $\gcd(240, 46)$ in the form of a Bézout identity.
5. Given the Bézout identity $\gcd(13, 6) = 13 \cdot 1 + 6 \cdot (-2)$, find multiplicative inverse of 6 modulo 13.
6. Find multiplicative inverse of 2 modulo 13 using the Euler's formula.
7. Calculate $6^{812} \bmod 13$ using square-and-multiply method. How many multiplications did you use? Describe the algorithm step-by-step.
8. Solve for x:
$$\begin{cases} x \equiv 2 \bmod 3 \\ x \equiv 3 \bmod 4 \end{cases}$$
9. Solve for x:
$$\begin{cases} x \equiv 1 \bmod 2 \\ x \equiv 2 \bmod 3 \\ x \equiv 3 \bmod 5 \end{cases}$$
10. Prove by induction that for all $n \in \mathbb{N}, n \geqslant 1$:
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \ .$$