

# Data Mining, Lecture 6

## Association Pattern Mining

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# What Pattern is?

- Pattern recognition is the discipline whose goal is the classification of objects into a number of classes or categories. [S.Theodoridis]
- What Pattern is? Object? Sub set?

# Market basket data

- Most popular example is *Supermarket data*. The goal is to determine *associations* between groups of items bought by customers.
- Discovered sets of items are referred to as *large itemsets*, *frequent itemsets*, or *frequent patterns*.
- Main applications include supermarket data (or shopping basket data in general), text mining, generalization to dependency-oriented data types.
- Within this chapter initial data will be referred as *transactions* and outputs as *itemsets*.

# The Frequent Pattern Mining Model

- Let  $U$  be the  $d$  - dimensional universe of elements (goods offered by the supermarket) and  $\mathcal{T}$  is the set of transactions  $T_1, \dots, T_n$ . They said that transaction  $T_i$  is drawn on universe of items  $U$ .
- $T_i$  may be represented by  $d$ -dimensional binary record.
- *itemset* is the set of items. *k-itemset* is the itemset containing exactly  $k$ -items.

# The Frequent Pattern Mining Model

## Definition

**Support** *The support of an itemset  $I$  is defined as the fraction of the transactions in the database  $\mathcal{T} = \{T_1, \dots, T_n\}$  that contain  $I$  as the subset*

The support of the itemset  $I$  is defined by  $sup(I)$ . Not to be confused with supremum.

## Definition

*Frequent Itemset Mining Given a set of transactions  $\mathcal{T} = \{T_1, \dots, T_n\}$  where each transaction  $T_i$  is drawn on the universe of elements  $U$ , determine all itemsets  $I$  that occur as a subset of at least a predefined fraction  $minsup$  of the transactions in  $\mathcal{T}$ .*

Predefined fraction  $minsup$  is referred as *minimal support*.

## Example: Market basket data set

<b>tid</b>	Set of items	Biary representation
1	{ Bread, Butter, Milk }	110010
2	{ Eggs, Milk, Yogurt }	000111
3	{ Bread, Cheese, Eggs, Milk }	101110
4	{ Eggs, Milk, Yogurt }	000111
5	{ Cheese, Milk, Yogurt }	001011

# The Frequent Pattern Mining Mode

## Definition

*Frequent Itemset Mining: Set-wise Given as set of sets  $\mathcal{T} = \{T_1, \dots, T_n\}$ , where each transaction  $T_i$  is drawn on the universe of elements  $U$ , determine all sets  $I$  that occur as the subset of at least a predefined  $\text{fractonminsup}$  of the sets in  $\mathcal{T}$ .*

**Support Monotonicity Property** *The support of every subset  $J$  of  $I$  is at least equal to the of the support of itemset  $I$ .*

$$\text{sup}(J) \geq \text{sup}(I) \quad \forall J \subset I$$

**Downward Closure Property** *Every subset of the frequent itemset is also frequent.*

## Definition

**Maximal Frequent Itemsets** *A frequent itemset is maximala at a given minimum support level  $\text{minsup}$ , if it is frequent and no superset of its frequent.*

# Association Rule Generation Framework

**Informal definition** If the presence of item set  $X$  in the certain transaction(s) leads (implies) presence of the set of items  $Y$  in the same transaction(s) then we talk about rule ( $X \Rightarrow Y$ ).

## Definition

**Confidence** Let  $X$  and  $Y$  be two sets of items. The confidence of the rule  $\text{conf}(X \Rightarrow Y)$  conditional probability of  $X \cup Y$  occurring in a transaction, given that the transaction contains  $X$

$$\text{conf}(X \Rightarrow Y) = \frac{\text{sup}(X \cup Y)}{\text{sup}(X)}$$

## Definition

**Association Rule** Let  $X$  and  $Y$  be two sets of items. Then, the rule  $X \Rightarrow Y$  is said to be an association rule at a minimum support of  $\text{minsup}$  and minimum confidence  $\text{minconf}$  if it satisfies following conditions.

- 1  $\text{sup}(X \cup Y) \geq \text{min sup}$
- 2  $\text{conf}(X \Rightarrow Y) \geq \text{minconf}$



# Frequent Itemset Mining Algorithms

- Brute force algorithms.
- The Apriori algorithm.
- Enumeration-Tree Algorithms
- Recursive Suffix-Based Pattern Growth Methods

# The Apriori Algorithm

**begin**

$k = 1;$

$\mathcal{F}_1 = \{ \text{All Frequent 1-itemsets} \};$

**while**  $\mathcal{F}_k \neq \emptyset$

    Generate  $\mathcal{C}_{k+1}$  by joining itemset-pairs in  $\mathcal{F}_k$ ;

    Prune itemsets from  $\mathcal{C}_{k+1}$  that violate downward closure;

    Determine  $\mathcal{F}_{k+1}$  by support counting on  $(\mathcal{C}_{k+1}, T)$  and  
    retaining from  $\mathcal{C}_{k+1}$  with support of at least minsup;

$k = k + 1;$

**end**

**return**  $(\cup_{i=1}^k \mathcal{F}_i)$

**end**

# Alternative Models: Interesting Patterns

- Collective strength
- Statistical Coefficient of Correlation
- $\chi^2$  Measure
- Nonlinear relationships

## Collective strength

- An itemset is said to be in *violation* of transaction, if some of the items are present in the transaction and others are not.
- The *violation rate*  $v(I)$  of the itemset  $I$  is defined as the fraction of violations of the itemset  $I$  over all transactions.
- The collective strength  $C(I)$  of the itemset  $I$  is defined as follows

$$C(I) = \frac{1 - v(I)}{1 - E[v(I)]} \cdot \frac{R[v(I)]}{v(I)}.$$

- The expected value of the  $v(I)$

$$R[v(I)] = 1 - \prod_{i \in I} p_i - \prod_{i \in I} (1 - p_i)$$

where  $p_i$  is the fraction of transactions where the item  $i$  occurs.

# Collective strength

- Let us consider *violation* to be an unfavorable event (prospective of establishing a high correlation among items)
- Collective strength may be expressed as follows:

$$C(I) = \frac{\text{Good events}}{E[\text{Good events}]} \frac{E[\text{Bad events}]}{\text{Bad events}}$$

- This leads us to the idea of *Negative Pattern Mining*. Determine patterns between the items or their absence.

# Statistical Coefficient of Correlation

Covariance is the measure of the strength of correlation between two sets of random variables.

$$\text{cov}(X, Y) = \sum_{i=1}^N \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}$$

Correlation coefficient is standardized

$$\rho_{XY} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$

or in another form

$$\rho = \frac{E[XY] - E[X]E[Y]}{\sigma(X)\sigma(Y)}$$

# Statistical Coefficient of Correlation

The Pearson correlation coefficient

$$\rho = \frac{E[XY] - E[X]E[Y]}{\sigma(X)\sigma(Y)}$$

May be rewritten in terms of *support* as follows

$$\rho_{ij} = \frac{sup(\{i, j\}) - sum(i) \cdot sup(j)}{\sqrt{sup(i) \cdot sup(j) \cdot (1 - sup(i)) \cdot (1 - sup(j))}}$$

Should we talk here about regression?

## $\chi^2$ measure

$\chi^2$  test allows to assess if unpaired observations of two categorical variables are independent of each other or not.

$$\chi^2 = \sum_{i=1}^{\nu_1 \cdot \nu_2} \frac{(\mathcal{O}_i - E_i)^2}{E_i}$$

where  $\nu_1$  and  $\nu_2$  are the degrees of freedom (number of categories) in the first and in second variables respectively. In the case of binary data  $\nu_1 \cdot \nu_2 = 2^{|X|}$ .



# Nonlinear

- $$y(x) = a_1x^n + a_2x^{n-1} + \dots + a_nx + b$$

- $$y(x) = f(x)$$

where  $f(\cdot)$  is arbitrary nonlinear function